

10.10 Logic

1 iii) $f(x, y) = x + y + x \cdot y \pmod{2}$

x	y	$f(x, y)$
1	1	1
1	0	1
0	1	1
0	0	0

2 i) $f(x, y) = \overline{xy}$

x	y	$f(x, y)$
1	1	0
1	0	1
0	1	1
0	0	1

3 ii)

A	B	$A \Rightarrow B$	$(\neg A) \vee B$	$(A \Rightarrow B) \Leftrightarrow ((\neg A) \vee B)$
T	T	T	T	T
F	T	T	T	T
T	F	F	F	T
F	F	T	T	T

The expression is a tautology.

3 i)

A	B	$A \Rightarrow B$	$(A \Rightarrow B) \Rightarrow B$	$((A \Rightarrow B) \Rightarrow B) \Rightarrow B$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

This is neither a tautology nor a contradiction.

5 i)

A	B	$(\neg A) \vee B$	$(\neg B) \vee A$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Since $(\neg A) \vee B$ has a different truth table to $(\neg B) \vee A$ the formulae is not logically equivalent.

- 6) ~~C~~ : Murphy is a Communist
A : Murphy is an atheist

$$((C \Rightarrow A) \wedge A) \Rightarrow C$$

A	C	$C \Rightarrow A$	$(C \Rightarrow A) \wedge A$	$((C \Rightarrow A) \wedge A) \Rightarrow C$
T	T	T	T	T
T	F	T	F	F
F	T	F	F	T
F	F	T	F	T

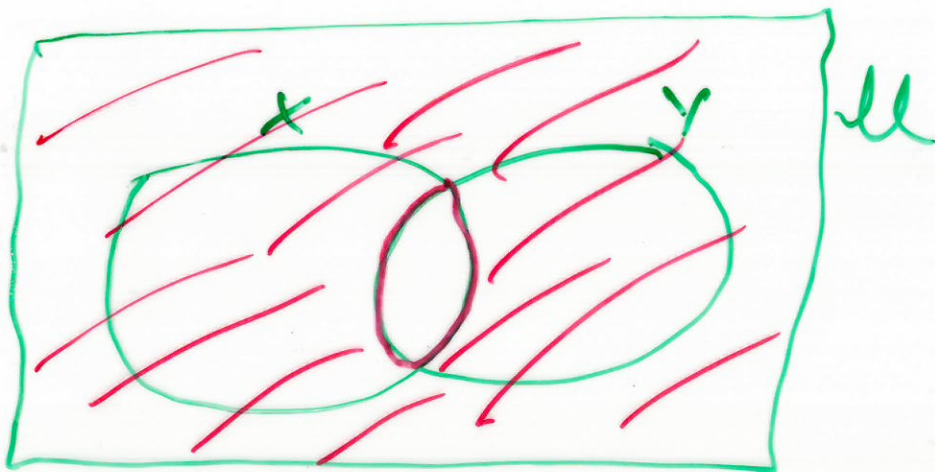
The argument is not logically valid because $((C \Rightarrow A) \wedge A) \Rightarrow C$ is not a tautology.

~~$x \wedge y$~~

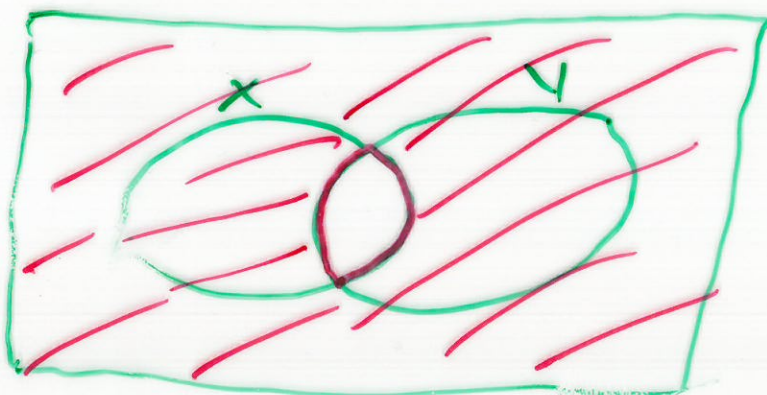
x	y	$\neg (x \wedge y)$	$(\neg x) \vee (\neg y)$
T	T	F	F
F	T	T	T
T	F	T	T
F	F	T	T

De Morgan's Law:

$$\neg (x \wedge y) = (\neg x) \vee (\neg y)$$



$$(x \cap y)^c$$



$$(x^c) \cup (y^c)$$