

Problem Solve

$$x + 2y + 3z = 1$$

$$2x + 5y + 5z = 2 \quad (*)$$

$$3x + 8y + 6z = 3$$

Soln Express (*) as

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}}_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

The solution is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Q. How do we find A^{-1} ?

Answer:

Gauss - Jordan method for finding the inverse of an $n \times n$ matrix A .

$$(A \mid I) \xrightarrow[\text{operations}]{\text{row}} (I \mid B)$$

Fact: $B = A^{-1}$.

There are three kinds of row operation:

(I) $R_i \mapsto R_i + \lambda R_j \quad (i \neq j, \lambda \in \mathbb{R})$

(II) $R_i \longleftrightarrow R_j$

(III) $R_i \rightarrow \lambda R_i \quad (\lambda \neq 0)$

Let's illustrate this with the above matrix A.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array} \right) \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{array} \right) \quad R_3 \rightarrow R_3 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array} \right) \quad R_3 \rightarrow -1 \cdot R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right) \quad \begin{array}{l} R_2 \rightarrow R_2 + R_3 \\ R_1 \rightarrow R_1 - 3R_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right) \quad R_1 \rightarrow R_1 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -12 & 5 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right)$$

$\underbrace{\hspace{10em}}_{A^{-1}}$

$$A^{-1} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}.$$

The solution to (*) is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Soln

$$x = 1$$

$$y = 0$$

$$z = 0.$$