

Matrix Notation

The linear system

$$x + 2y + 3z = 10$$

$$2x + 5y + 5z = 21$$

$$3x + 8y + 6z = 31$$

can be written more succinctly

as:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix} \quad (**)$$

↑
3x3 matrix
3 rows
3 columns

↑
3x1 matrix

An m x n matrix is an array of numbers with m rows and n columns.

e.g. $(1 \ 2 \ 3)$ is a 1×3 matrix

Terminology:

An $m \times 1$ matrix is called a Column Vector

A $1 \times n$ " " " row Vector

A row vector of length n

$$R = (a \ b \ c)$$

and a column vector of length n

$$C = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

can be multiplied using the definition

$$RC = \overset{1 \times 3}{(a \ b \ c)} \overset{3 \times 1}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} \overset{1 \times 1}{=} ax + by + cz$$

Example

$$(1 \ 2 \ 3) \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 32$$

Two 3×3 matrices can be multiplied as follows.

Suppose

$$A = \left(\begin{array}{c} \hline R_1 \\ \hline R_2 \\ \hline R_3 \end{array} \right)$$

$$B = \left(\begin{array}{c|c|c} c_1 & c_2 & c_3 \end{array} \right)$$

Then

$$AB = \begin{pmatrix} R_1 c_1 & R_1 c_2 & R_1 c_3 \\ R_2 c_1 & R_2 c_2 & R_2 c_3 \\ R_3 c_1 & R_3 c_2 & R_3 c_3 \end{pmatrix}$$

Example

$$A = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 3 & 6 \end{pmatrix}$$

Then

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Remark For $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

we have

$$IA = A = AI$$

for any 3×3 matrix A .

we call I the identity
matrix

our original system (**)
can be written

$$B \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

"So"

$$A B \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

"So"

$$I \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

So

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

Hence $x=3, y=2, z=1$.