Exercise Solve the linear system

$$x + y + z = -2$$
$$3x + 3y - z = 6$$
$$x - y + z = -1$$

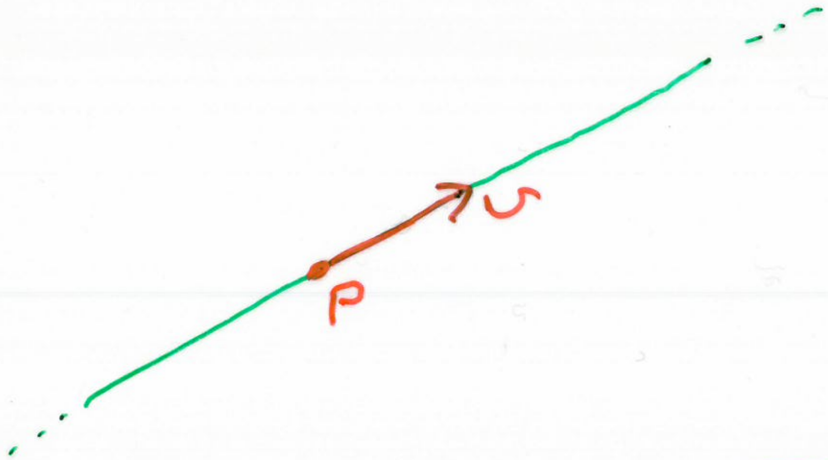
$$x + y + z = -2$$
$$-4z = 12 \qquad ② \mapsto ② - 3①$$
$$-2y = 1 \qquad ③ \mapsto ③ - ①$$

$$x + y + z = -2$$
$$-2y = 1 \qquad ② \leftrightarrow ③$$
$$-4z = 12$$

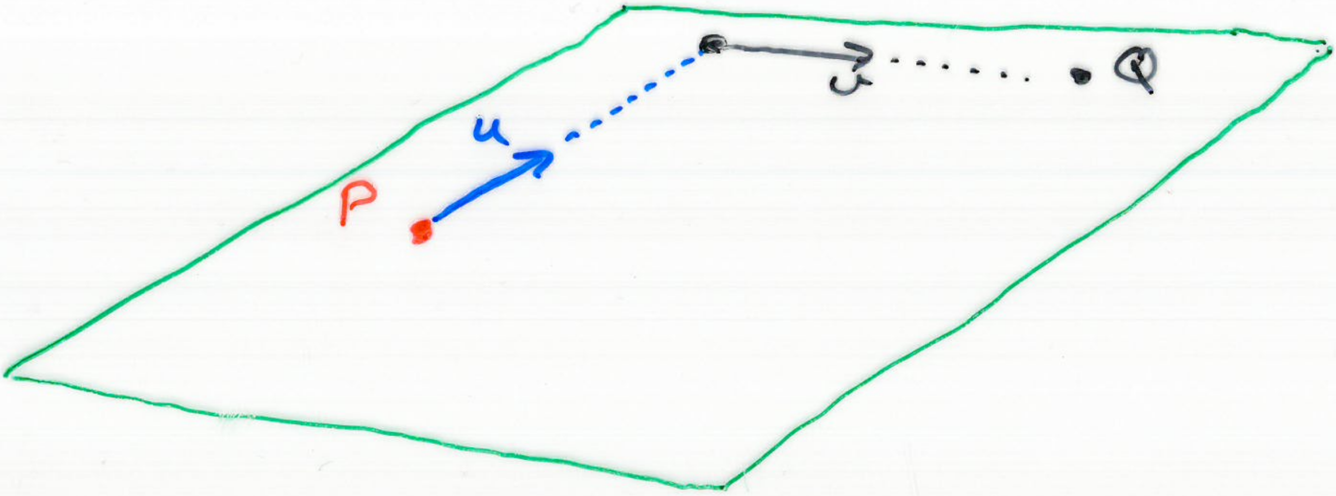
Back substitution

$$z = -3$$
$$y = -\frac{1}{2}$$
$$x = \frac{3}{2}.$$

A line (in $\mathbb{R}^3$) is determined by a point P on the line, and a direction vector.



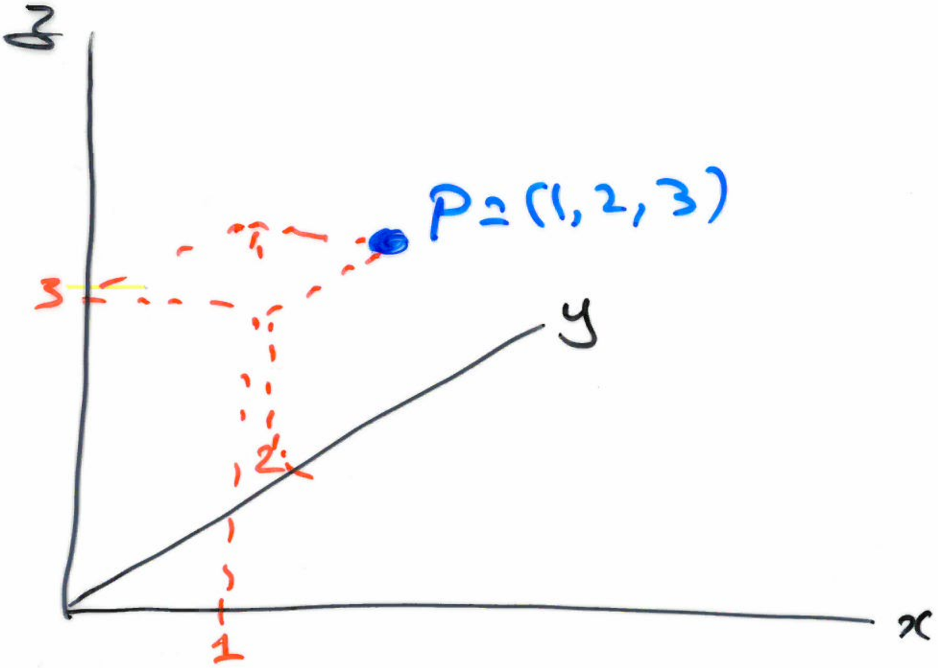
A plane (in $\mathbb{R}^3$) is determined by a point P and two direction vectors



in $\mathbb{R}^3$ a point is a triple

$$p = (1, 2, 3)$$

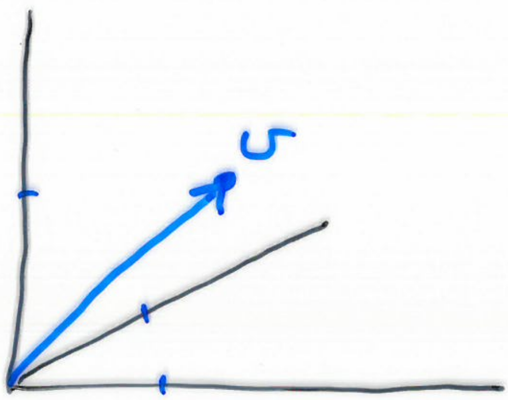
for instance.



A direction vector v in $\mathbb{R}^3$ is also a triple

$$v = (1, 1, 2)$$

for instance.



A point Q in the plane has the form

$$Q = P + \lambda u + \mu v$$

where $\lambda, \mu \in \mathbb{R}$.

<u>Example</u> Let

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 3x + 5y + 7z = 15 \right\} \subseteq \mathbb{R}^3$$

Note: $P = \begin{pmatrix} \frac{15}{3} \\ 0 \\ 0 \end{pmatrix}$ lies in S.

An arbitrary point Q is S has the form

$$Q = \begin{pmatrix} 5 - \frac{5}{3}\lambda - \frac{7}{3}\mu \\ \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{5}{3} \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -\frac{7}{3} \\ 0 \\ 1 \end{pmatrix}$$

So S is in fact the plane determined by

$$p = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \text{ and directions}$$

$$u = \begin{pmatrix} -\frac{5}{3} \\ 1 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} -\frac{7}{3} \\ 0 \\ 1 \end{pmatrix}.$$

Conclusion:

~~Any equation~~

A linear equation such as $3x + 5y + 7z = 15$ determines a plane in $\mathbb{R}^3$