Systems of Linear Equations

systems of linear equations

Matrix algebra

- very algebraic
- very computational

Vector spaces

- geometric insight

Example

A factory requires energy, steel and labour to manufacture three machines A, B, C.

Resource	A	B	C	weekly available
Energy	2 MWh	3 MWh	2 MWh	100 MWh
steel	1 tonne	1 tonne	4 tonnes	70 tonnes
labour	20 h	10 h	10 h	500 hours

what production figures ensure all resources are used?

Sol^n Let's suppose we manufacture

x units of machine A

y " " " B

z " " " C

If all resources are to be used then x, y, z must satisfy:

$$
\begin{aligned}
① \quad & \boxed{2}x + 3y + 2z = 100 \\
② \quad & x + y + 4z = 70 \\
③ \quad & 20x + 10y + 10z = 500
\end{aligned}
$$

system of linear equations.

$$\begin{cases} 2x + 3y + 2z = 100 & \\ -\frac{1}{2}y + 3z = 20 & ② \mapsto ② - \frac{1}{2} ① \\ -20y - 10z = -500 & ③ \mapsto ③ - 10 ① \end{cases}$$

This system of linear equations has the same solutions as the first system. But it is a bit simpler.

$$\begin{cases} 2x + 3y + 2z = 100 & \\ \boxed{-\frac{1}{2}}y + 3z = 20 & \\ -130z = -1300 & ③ \mapsto ③ - 40 ② \end{cases}$$

Back substitution gives

$z = 10$

$y = 20$

$x = 10$

This procedure is known as Gaussian elimination.

Q) When could this general procedure fail?

Terminology
2 is the pivot at the first stage
$-\frac{1}{2}$ is the pivot at the second stage

A) The procedure could fail if one of the pivots is zero.