yesterday:

$$\boxed{\begin{aligned} |zw| &= |z|\,|w| \\ \operatorname{Arg}(zw) &= \operatorname{Arg}(z) + \operatorname{Arg}(w) \end{aligned}}$$

Should also mention

$$\boxed{\begin{aligned} \left|\frac{z}{w}\right| &= \frac{|z|}{|w|} \\ \operatorname{Arg}\left(\frac{z}{w}\right) &= \operatorname{Arg}(z) - \operatorname{Arg}(w) \end{aligned}}$$

so

$$|z^2| = |z|^2$$

$$|z^3| = |z|^3$$

$$\vdots$$

$$\boxed{|z^n| = |z|^n \quad \text{for } n \in \mathbb{Z}}$$

$$\operatorname{Arg}(z^2) = 2\operatorname{Arg}(z)$$

$$\operatorname{Arg}(z^3) = 3\operatorname{Arg}(z)$$

$$\vdots$$

$$\boxed{\operatorname{Arg}(z^n) = n\operatorname{Arg}(z) \quad \text{for } n \in \mathbb{Z}}$$

De Moivre's Theorem

Problem Evaluate

$$\left(\frac{1+i}{1-i}\right)^{10}$$

Sol^n

$$\left(\frac{1+i}{1-i}\right)^{10} = \left(\frac{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}{\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{\pi}{4}\right)}\right)^{10}$$

$$= \frac{\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{10}}{\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)^{10}}$$

$$= \frac{\cos\frac{10\pi}{4}+i\sin\frac{10\pi}{4}}{\cos\frac{70\pi}{4}+i\sin\frac{70\pi}{4}}$$

$$= \frac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{4}}$$

$$= \cos(-\pi)+i\sin(-\pi)$$

$$= -1$$

<u>Problem</u> Find all z such that $z^4 = 1$. Then factorize the polynomial

$$x^4 - 1$$

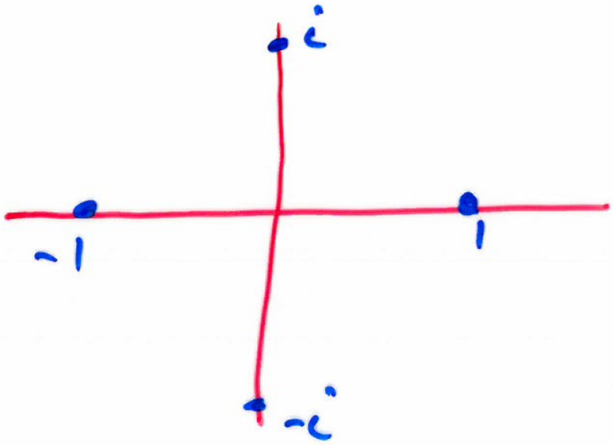
as

i) a product of linear factors

ii) a product of real linear/quadratic factors.

<u>Sol^n</u>

$$1^4 = 1$$
$$(-1)^4 = 1$$
$$i^4 = 1$$
$$(-i)^4 = 1$$



$$p(x) = x^4 - 1$$

$p(1) = 0$

$p(-1) = 0$

$p(i) = 0$

$p(-i) = 0$

so

$$x^4 - 1 = (x-1)(x+1)(x-i)(x+i)$$

or

$$\boxed{x^4 - 1 = (x-1)(x+1)(x^2+1)}$$

Problem Find all z such that $z^5 = 1$.

Then factorise the polynomial

$$x^5 - 1$$

as

i) a product of linear factors

ii) a product of real linear/quadratic factors.

Soln

need $\operatorname{Arg}(z^5) = \operatorname{Arg}(1)$

$$\boxed{5\operatorname{Arg}(z) = 0}$$

need $|z^5| = 1$

or $|z|^5 = 1$

$$\boxed{\text{or } |z| = 1.}$$

possibilities for z are :

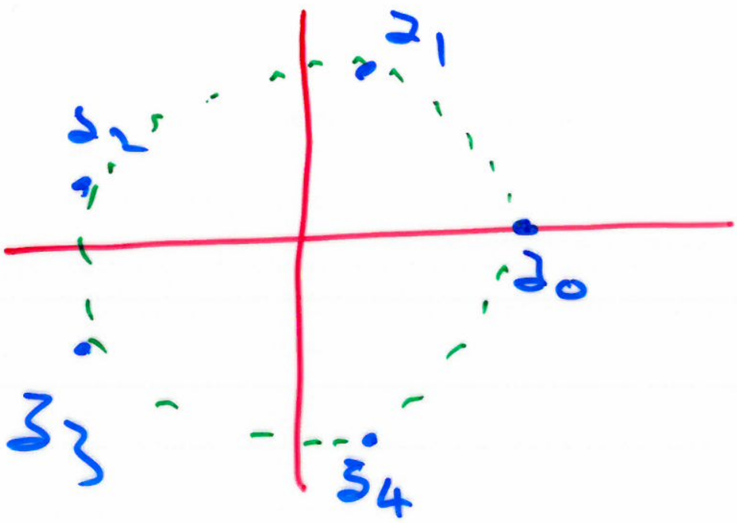
$$z_0 = 1 = \cos 0 + i \sin 0$$

$$z_1 = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$$

$$z_2 = \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$$

$$z_3 = \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$$

$$z_4 = \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{4}$$



$$x^5 - 1 = (x-1)(x-z_1)(x-z_2)(x-z_3)(x-z_4)$$