

Algebra

- ① Logic (8 lectures)
- ② systems of equations (8 lectures)
- ③ complex numbers.

Logic

Is the following argument
logically valid?

Indeed, what do we mean
by "logically valid"?

If fallout shelters are built, other countries will feel endangered and our people will get a false sense of security. If other countries will feel endangered they may start a preventative war. If our people will get a false sense of security, they will put less effort into preserving peace. If fallout shelters are not built, we run the risk of tremendous losses in the event of war. Hence, either other countries may start a preventative war and our people will put less effort into preserving peace, or we run the risk of tremendous losses in the event of war.

To start studying logic let's
recall clock arithmetic.

$$7 + 8 \equiv 3 \pmod{12}$$

$$7 - 8 \equiv -1 = 11 \pmod{12}$$

$$7 \times 8 = 8 \pmod{12}$$

$$7^{-1} \times 8 = \pmod{12}$$

$$7^{-1} = ?$$

$$7 \times 7 = 1 \pmod{12}$$

So 7^{-1} is that number
which when multiplied
by 7 gives 1.

$$\text{So } 7^{-1} = 7.$$

$$7^{-1} \times 8 = 7 \times 8 = 8 \pmod{12}.$$

From now on let's work modulo 2.

$$1 \times 1 = 1 \quad \text{mod } 2$$

$$1 \times 0 = 0 \quad \text{mod } 2$$

$$1 + 1 = 0 \quad \text{mod } 2$$

$$1 + 0 = 1 \quad \text{mod } 2$$

Example Consider the function

$$f(x, y) = xy + x + y \quad \text{mod } 2$$

Let's evaluate $f(x, y)$ for all possible x, y .

x	y	$xy + x + y \quad \text{mod } 2$
1	1	1
1	0	1
0	1	1
0	0	0

But how is this in any way related to logic?

P_1 : Graham Ellis is Welsh. T (true)

P_2 : Graham Ellis is French. F (false)

Q_1 : Cardiff is the capital of Wales. T

Q_2 : Cardiff is the capital of Ireland. F

P_1 or Q_1 : Graham Ellis is
Welsh or Cardiff
is the capital
of Wales. T

P_1 or Q_2 : Graham Ellis is
Welsh or
Cardiff is the
capital of Ireland T

P_2 or Q_2 : Graham Ellis is
French or
Cardiff is the
capital of Ireland. F

The meaning of "or" is described by the following table.

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

x	y	$xy + x + y \pmod 2$
1	1	1
1	0	1
0	1	1
0	0	0