

Complex Variables MA287 : Third & Final Test

Time allowed: 50 minutes

Attempt all five questions

**Name & Id. :**

Q1. [5 marks]

Evaluate

$$\int_C \frac{e^z}{z-2} dz$$

where  $C$  is the circle  $|z| = 3$  with positive direction.

Q2. [5 marks]

Evaluate

$$\int_C \frac{e^{3z}}{z - \pi \mathbf{i}} dz$$

where  $C$  is the ellipse  $|z-2| + |z+2| = 6$  with positive direction.

Q3. [5 marks]

Using the definition of a limit, prove that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right) = 1$$

for all  $z \in \mathbb{C}$ .

Q4. [5 marks]

We have seen in lectures that

$$\log(1+z) = z - z^2/2 + z^3/3 - z^4/4 + \dots$$

for  $|z| < 1$ . Now expand

$$\log\left(\frac{1+z}{1-z}\right)$$

in a Taylor series about  $z = 0$ .

Q5. [5 marks]

Find the residues of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

at all poles in the finite plane. Evaluate

$$\int_{\Gamma} \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz$$

where  $\Gamma$  is the circle  $|z - 3\mathbf{i}| = 3/2$  with positive orientation.