

Complex Variables MA287 : First Test

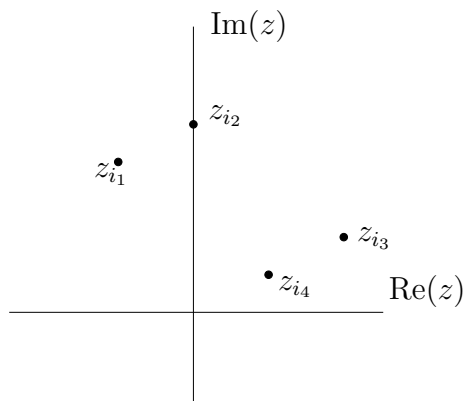
Time allowed: 50 minutes

Attempt all five questions

Name & Id. :

Q1. [5 marks]

Four complex numbers  $z_1, z_2, z_3, z_4$  are shown, to scale, in the complex plane. They satisfy the following conditions:  $z_2 = \mathbf{i}z_1$ ,  $z_3 = kz_1$  where  $k \in \mathbb{R}$ ,  $z_4 = z_2 + z_3$ .



Determine  $i_1, i_2, i_3, i_4$  and estimate  $k$ .

Q2. [5 marks]

Describe and graph the locus represented by  $|z + 2\mathbf{i}| + |z - 2\mathbf{i}| = 6$ .

Q3. [5 marks]

Prove that  $\sin^2 z + \cos^2 z = 1$ .

Q4. [5 marks]

Describe a Riemann surface  $S$  for the function  $f(z) = (z^2 + 1)^{\frac{1}{3}}$  by determining: (i) a branch point, (ii) a branch line, (iii) the number of copies of  $\mathbb{C}$  that are glued together, and the fashion in which they are glued.

Q5. [5 marks]

Let  $f(z) = (z^2 + 4)/(z - 2\mathbf{i})$  for  $z \neq 2\mathbf{i}$  and  $f(2\mathbf{i}) = 3 + 4\mathbf{i}$ . Is  $f(z)$  continuous at  $z = 2\mathbf{i}$ ? Justify your answer.