1st class test: Monday 4th Feb.
30 min. test based on tutorial
sheets I & II.

Problem

Consider $f(z) = (z^2+1)^{\frac{1}{2}}$

a) Show that $+i$ and $-i$ are branch points of f.

b) choose a branch line(s).

c) Describe a Riemann surface S for which $f: S \to \mathbb{C}$ is a single valued continuous function.

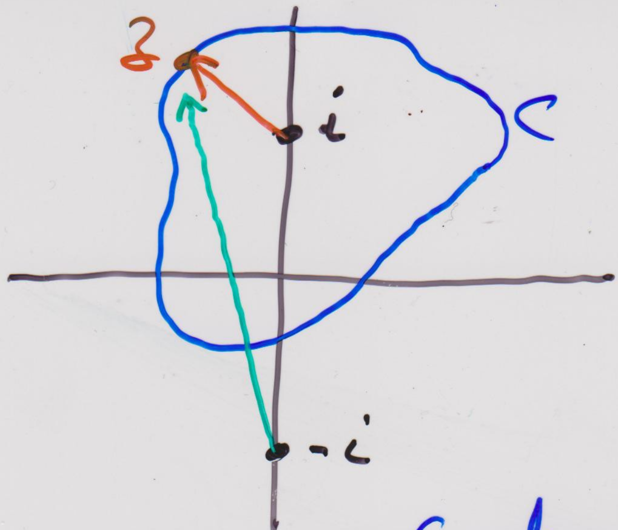
Sol^n

a) $w = (z^2+1)^{\frac{1}{2}} = ((z-i)(z+i))^{\frac{1}{2}}$

$\arg(w) = \frac{1}{2}\arg(z-i) + \frac{1}{2}\arg(z+i)$

change in $\arg(w)$ =

$\frac{1}{2}$ change in $\arg(z-i)$ + $\frac{1}{2}$ change in $\arg(z+i)$



Consider a curve C about i. As z move once around C

change $\arg(z-i) = 2\pi$

change $\arg(z+i) = 0$

so

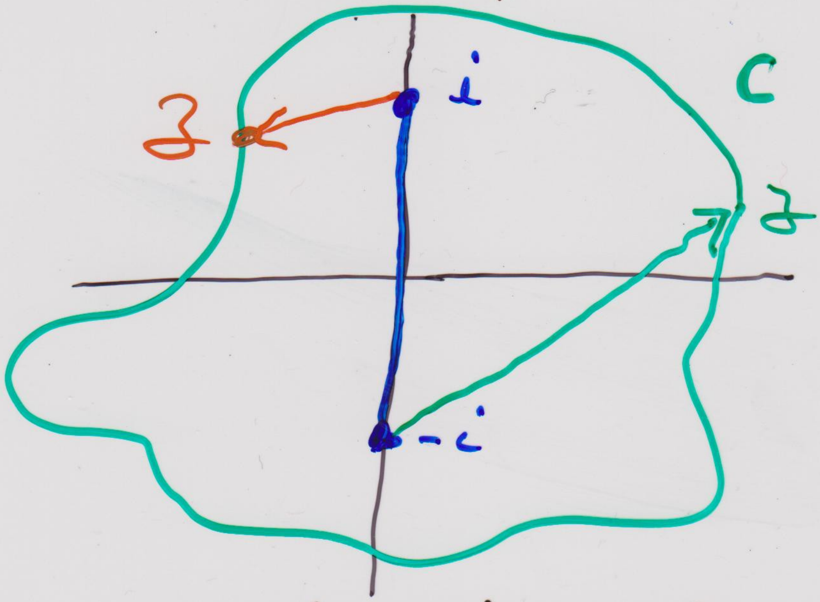
change in $\arg(w) = \frac{1}{2}(2\pi) + \frac{1}{2}(0) = \pi$

Thus, when z travels once around C, w does not return to its original value.

So $z=i$ is a branch point.

Similarly, $z=-i$ is a branch point,

) A suitable branch line is



To see that this is a branch line, choose any circuit C in ℂ which does not cross the line.

Travelling once around C produces the following

change in arg(w)

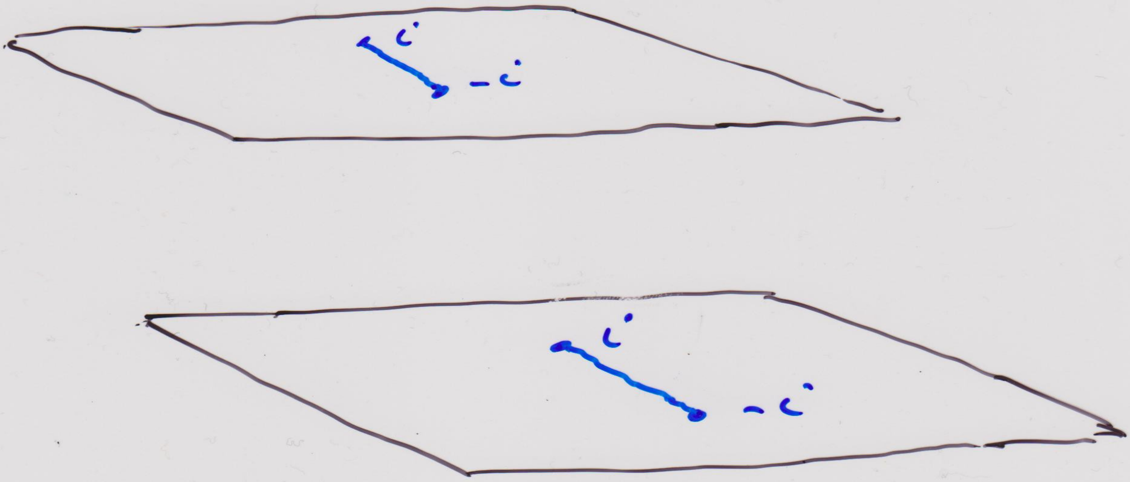
$= \frac{1}{2}$ change in arg$(z-i)$ + $\frac{1}{2}$ change in arg$(z+i)$

$= \frac{1}{2}(2\pi) + \frac{1}{2}(2\pi)$

$= 2\pi$.

Thus there is no change in $w = (z^2+1)^{\frac{1}{2}}$ if we travel once around such a curve C.

c) A suitable Riemann surface for $f(z)$ consists of two copies of the complex plane $\mathbb{C}$.



Each copy of $\mathbb{C}$ is cut along its branch line; opposite edges of the cut are joined.