Tutorials

Tuesday 6 - 7 Dillon

Thursday 2 - 3 ??

Tutor: Le Van Luyen

Theorem There are precisely n n^{th} roots of unity and they sum to 0.

Proof If $z^n = 1$ then

$$z = \omega_k = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$$

for $k = 0, 1, \dots, n-1$.

So there are precisely n n^{th} roots of unity.

Consider the polynomial

$$f(z) = z^n - 1 .$$

If $f(\omega_k) = 0$ then, by the Euclidean algorithm (MA130),

we find

$$f(z) = (z - \omega_k)\, g(z)$$

with $g(z)$ a polynomial of degree $n-1$.

By induction

$$\begin{array}{l} f(z) = (z-\omega_0)(z-\omega_1)\cdots(z-\omega_{n-1}) \\ \parallel \\ z^n - 1 \end{array}$$

Thus

$$0\, z^{n-1} = -(\omega_0 + \omega_1 + \cdots + \omega_{n-1})\, z^{n-1}.$$

Hence

$$\omega_0 + \omega_1 + \cdots + \omega_{n-1} = 0$$

$\boxed{\text{MY}}$

Problem Prove that

$$\cos\frac{2\pi}{6}+\cos\frac{4\pi}{6}+\cos\frac{6\pi}{6}+\cos\frac{8\pi}{6}+\cos\frac{10\pi}{6}=-1$$

Proof

Consider $z^6-1=0$, whose solutions are

$$\omega_k=\cos\frac{2k\pi}{6}+i\sin\frac{2k\pi}{6}$$

for $k=0,1,2,3,4,5$.

Now

$$\omega_0+\omega_1+\omega_2+\omega_3+\omega_4+\omega_5=0.$$

Equating real parts:

$$1+\frac{\cos 2\pi}{6}+\frac{\cos 4\pi}{6}+\frac{\cos 6\pi}{6}+\frac{\cos 8\pi}{6}+\frac{\cos 10\pi}{6}=0.$$

$\boxed{\text{ux}}$

Defn

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

And for $z = x + iy$

$$\boxed{e^{z} = e^{x} e^{iy} = e^{x}(\cos y + i \sin y)}$$

Observe:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\Rightarrow \quad e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

similarly

$$\boxed{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

Defn The complex conjugate of $z = x + iy$ is

$$\bar{z} = x - iy.$$

For instance, the circle

$$x^2 + y^2 = 25$$

can be expressed as:

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$\frac{1}{2}(z + \bar{z}) = x$$

$$\frac{1}{2i}(z - \bar{z}) = y$$

and

$$x^2 + y^2 = 25$$

$$\frac{1}{2}(z+\bar{z})\frac{1}{2}(z+\bar{z}) +$$

$$\frac{1}{2i}(z-\bar{z})\frac{1}{2i}(z-\bar{z}) = 25$$

$$\frac{1}{4}(z^2+2z\bar{z}+\bar{z}^2)$$

$$-\frac{1}{4}(z^2-2z\bar{z}+\bar{z}^2) = 25$$

$$\boxed{z\bar{z} = 25}$$

The circle.