Lecture 9

Linear Algebra

study of linear equations

Matrix Algebra
- very computational

vector spaces
- geometric insight

Example

A factory requires energy, steel and labour to manufacture three machines A, B, C.

Resource	A	B	C	weekly available
Energy	2 MWh	3 MWh	2 MWh	100 MWh
Steel	1 tonne	1 tonne	4 tonne	70 tonnes
Labour	20 hrs	10 hrs	10 hrs	500 hrs

what production figures ensure all resources are used?

Sol^n

Let's suppose we manufacture

x units of machine A

y " " B

z " " C

If all resources are to be used, then:

$$\left.\begin{aligned} R1:&\ \boxed{2}x + 3y + 2z = 100 \\ R2:&\ x + y + 4z = 70 \\ R3:&\ 20x + 10y + 10z = 500 \end{aligned}\right\} \text{linear system}$$

This is equivalent to the system:

$$\begin{aligned} 2x + 3y + 2z &= 100 && R1 \\ \boxed{-\tfrac{1}{2}}y + 3z &= 20 && R2 - \tfrac{1}{2}R1 \\ -20y - 10z &= -500 && R3 - 10R1 \end{aligned}$$

N.B. This procedure is known as Gaussian elimination, and can be applied to n equations in n unknowns.

Question when could this general procedure fail?

Answer: if one of the "pivots" is zero.

2 is the pivot in the first stage
$-\frac{1}{2}$ " " second "

Exercise Solve the linear system

$$x + y + z = -2$$
$$3x + 3y - z = 6$$
$$x - y + z = -1$$

This is equivalent to

$$\begin{aligned} 2x + 3y + 2z &= 100 && R1 \\ -\tfrac{1}{2}y + 3z &= 20 && R2 \\ -130z &= -1300 && R3 - 40R2 \end{aligned}$$

Back substitution yields:

$\boxed{z = 10}$ R3

$-\frac{1}{2}y + 30 = 20$ R2

$\Rightarrow -\frac{1}{2}y = 20 - 30$

$\Rightarrow \frac{1}{2}y = 30 - 20$

$\Rightarrow y = 2.10$ $\boxed{y = 20}$

$2x + 60 + 20 = 100$ R1

$\Rightarrow$ $\boxed{x = 10}$