

Sets

A set is a collection of things.

$\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\}$ set of natural numbers.

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ set of integers

$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \right\}$ set of rational numbers.

\mathbb{R} = set of real numbers

$\{1, 2, 3, 4, 5\}$ set of five natural numbers.

Notation

$$42 \in \mathbb{N}$$

42 is a member of \mathbb{N}

$$\sqrt{2} \notin \mathbb{Q}$$

$\sqrt{2}$ is not rational

$$\mathbb{Z} \subseteq \mathbb{Q}$$

The integers are a subset of the rationals

$$\mathbb{Q} \not\subseteq \mathbb{Z}$$

The rationals are not a subset of the integers.

$$\{x \in \mathbb{Z} : -2 < x < 5\} = \{-1, 0, 1, 2, 3, 4\}$$

Translation

S is the set of integer multiples of 5.

$$S = \{ x \in \mathbb{Z} : x = 5k \text{ for some } k \in \mathbb{Z} \}$$

T is the set of perfect square integers.

$$T = \{ m^2 : m \in \mathbb{Z} \}$$

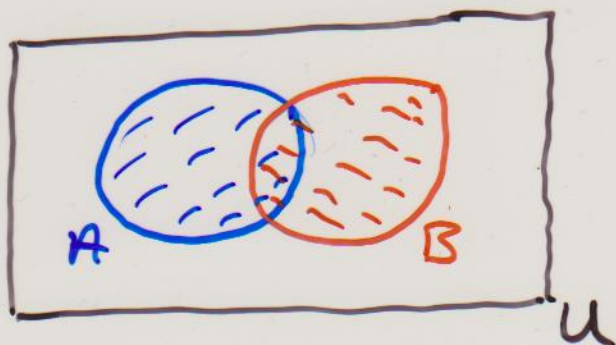
Operations on sets

Let U be a set (the universal set)

Let $A \subseteq U$, $B \subseteq U$.

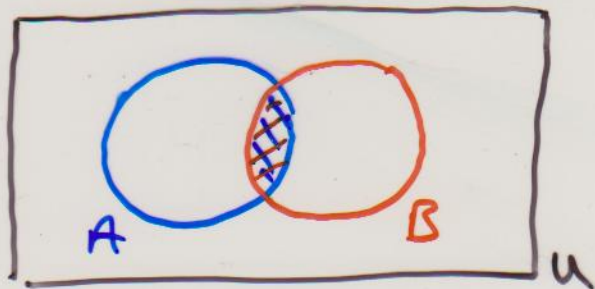
Defn "Union"

$$A \cup B = \{ x \in U : x \in A \text{ or } x \in B \}$$



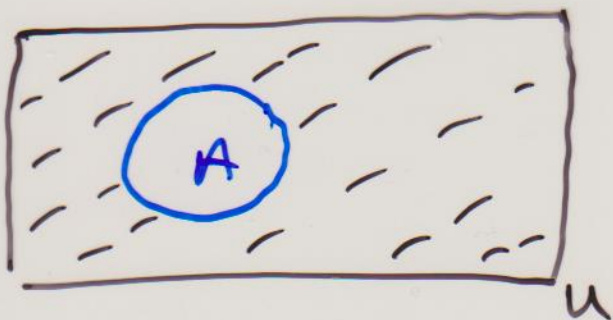
Defn "Intersection"

$$A \cap B = \{x \in U : x \in A \text{ and } x \in B\}$$



Defn "Complement"

$$A' = \{x \in U : x \notin A\}$$



Examples $U = \mathbb{N}$

$$B = \{2, 4, 6, 8, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$C = \{8, 10, 12\}$$

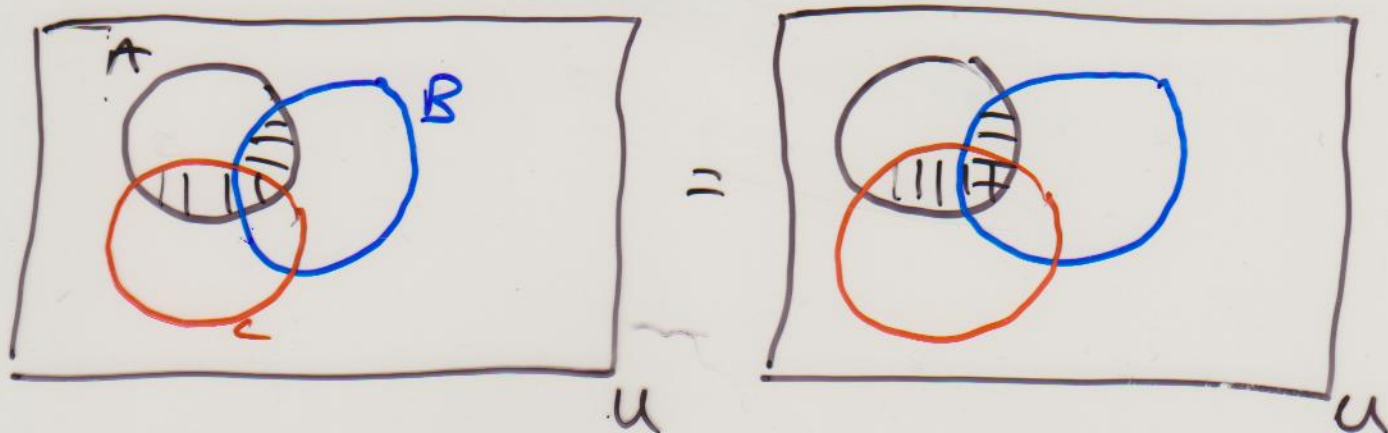
$$A \cap B = \{2, 4\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

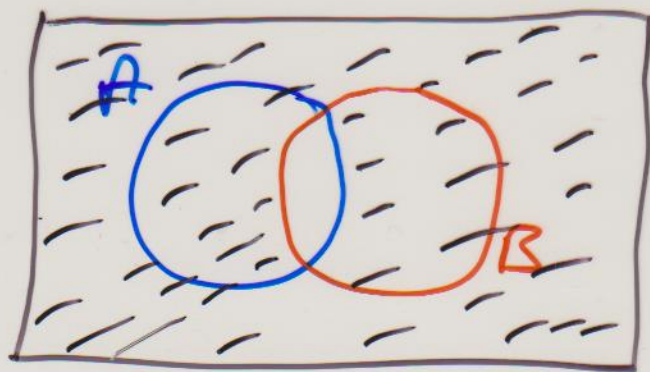
$$A \cap C = \emptyset \quad \text{empty set.}$$

Properties and Venn diagrams

Fact: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



Fact: $(A \cap B)' = A' \cup B'$



Power sets

$P(A)$ = the set of all subsets of A .

Illustration

$$A = \{1, 2, 3\}$$

$$P(A) =$$

$$\{ \{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \\ \{1\}, \{2\}, \{3\}, \emptyset \}$$

