1. First homework deadline postponed by 1 week

2. To find MA160/MA190 algebra notes:

Google "Graham Ellis" "NUIG" then click on teaching page

or

http://hamilton.nuigalway.ie

Predicates

$p(x)$: x is greater than 5

$p(2)$ False

$p(6)$ True

<u>Definition</u> A <u>predicate</u> is a statement $p(x)$ depending on a variable x such that, for any particular value of x, the statement is either True or False.

Given two predicates $p(x)$, $q(x)$ we can build new predicates such as

$p(x) \wedge q(x)$

$q(x) \vee p(x)$

$\neg p(x)$

$p(x) \rightarrow q(x)$

Example

$p(x)$: $(x > 15) \wedge (x < 20)$

$p(16)$ True

$p(14)$ False

Negation (x, y range over all integers)

predicate	Negated predicate
$x > 5$	$x \leq 5$
$(x > 0) \wedge (x < 10)$	$(x \leq 0) \vee (x \geq 10)$
$\neg(x = 8)$	$x = 8$
$(x \geq 0) \vee (y \geq 0)$	$(x < 0) \wedge (y < 0)$

For the set of numbers

$$D = \{-1, 0, 1, 2\}$$

Complete the table:

predicate	True for which members of D	True for at least one	True for all
$x < 0$	-1	YES	NO
$x > -3$	$-1, 0, 1, 2$	YES	YES
$x^2 < x$	None	NO	NO
$x^2 \geq x$	$-1, 0, 1, 2$	YES	YES

Handy notation

$k \in D$ k is an element in D

$\forall$ for all

$\exists$ there exists

Let $D = \{-2, -1, 0, 1, 2\}$

statement: $\forall d \in D,\ d > -2$ FALSE

Negated statement: $\exists d \in D,\ d \leq -2$ TRUE

statement: $\exists m \in D,\ m > 10$ FALSE

negated statement $\forall m \in D,\ m \leq 10$ TRUE

Multiple Quantifiers

$p(x,y)$: $x \cdot y = 36$

$p(3,4)$ FALSE

$p(-4,-9)$ TRUE

There exist integers x and y such that $xy = 36$.

$\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z},\ xy = 36$ TRUE

Examples Translate, and then evaluate for $D = \{3, 4, 5, 10, 20, 25\}$

For every n in D, $n < 20$

$\forall n \in D, \ n < 20$ FALSE

For all n in the set D, $n < 5$ or n is a multiple of 5.

$\forall n \in D,$ $(n < 5)$ or n is multiple of 5.

TRUE

There is at least one k in D with the property that k^2 is in D.

$\exists \ k \in D$ such that $k^2 \in D$

TRUE

Negating quantified statements

For every choice of integers s,t it is true that $s^2+t^2 \geq 0$.

$\forall s,t \in \mathbb{Z},\ s^2+t^2 \geq 0$ TRUE

Statement: $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x+2y=3$

Negated statement: $\neg(\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x+2y=3)$

or $\exists x \in \mathbb{Z},\ \neg(\exists y \in \mathbb{Z},\ x+2y=3)$

or $\exists x \in \mathbb{Z},\ \forall y \in \mathbb{Z},\ \neg(x+2y=3)$

or better $\exists x \in \mathbb{Z},\ \forall y \in \mathbb{Z},\ x+2y \neq 3$

Statement: $\exists x>0,\ \forall y>0,\ xy<x$

Negated statement $\neg(\exists x>0,\ \forall y>0,\ xy<x)$

$\forall x>0,\ \neg(\forall y>0,\ xy<x)$

$\forall x>0,\ \exists y>0,\ \neg(xy>x)$

$\forall x>0,\ \exists y>0,\ xy \leq x$.