Valid arguments

Is the following argument valid?

If h don't get an A in the final exam h won't get an A for the course. h got an A for the course.

Therefore h got an A on the final exam.

Let's first translate the argument into logic.

f: h get an A on the final exam.

c: h get an A for the course.

$$((\neg f \rightarrow \neg c) \wedge c) \rightarrow f$$

Definition An argument is **valid** if the corresponding logical expression always has the value True.

f	c	$\neg f \rightarrow \neg c$	$(\neg f \rightarrow \neg c) \wedge c$	$((\neg f \rightarrow \neg c) \wedge c) \rightarrow f$
T	T	T	T	T
T	F	T	F	T
F	T	F	F	T
F	F	T	F	T

Conclusion: our original argument is valid.

Definition A logical formula is a **tautology** if it is always true.

e.g. $((\neg f \rightarrow \neg c) \wedge c) \rightarrow f$ is a tautology

Is the following valid ?

You must pass the final exam or pass all the tests if you pass the course. You failed the final exam and the course. Therefore you must not have passed all the tests.

f: you pass final exam

c: you pass the course

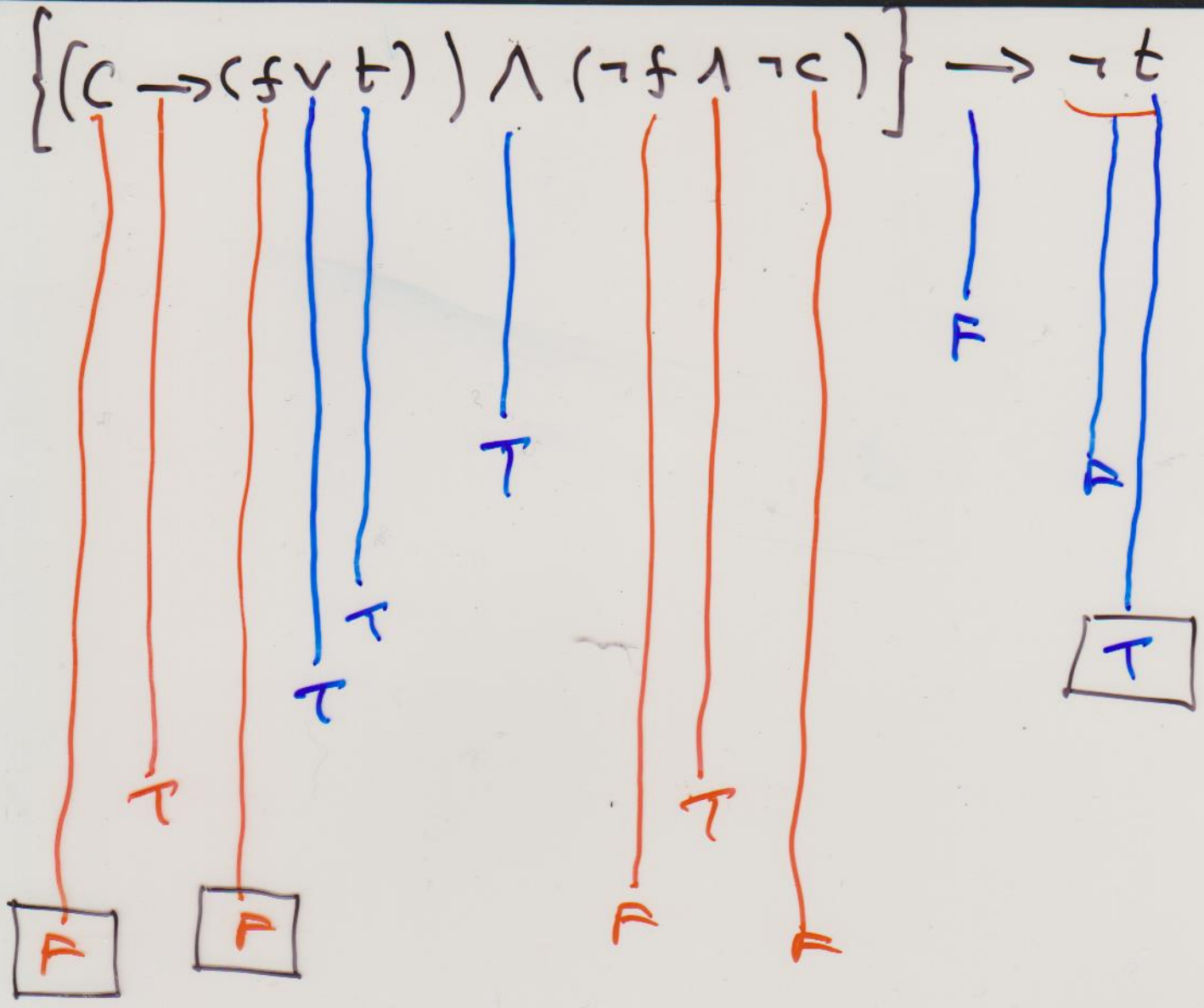
t: you pass all the tests

$$\big((c \to (f \vee t)) \wedge (\neg f \wedge \neg c)\big) \to \neg t$$

c f t	c → (f ∨ t)	¬f ∧ ¬c	(c → (f ∨ t)) ∧ (¬f ∧ ¬c)	final expression
T T T	T	F	F	T
T T F	T	F	F	T
T F T	T	F	F	T
F T T	T	F	F	T
T F F	F	F	F	T
F T F	T	F	F	T
F F T	T	T	T	F
F F F	T	T	T	T

So the final expression is not a tautology, and the original argument is not valid.

An alternative, and quicker, method to see that the expression is not a tautology is as follows.


{(C → (f ∨ t)) ∧ (¬f ∧ ¬C)} → ¬t
T
T
T
T
T
F
T
T
F
F
F
F