

1st homework now online.
Deadline: Friday week.

IF THEN

Riddle You meet two islanders.

A says "if B is truthful then so am I".

B says "at least one of us is lying".

Who, if anyone, is telling the truth?

We first need to understand the
truth table of

if p then q .

Example Are the following true.

1. "if $2+2=4$ then $3^2=9$ "
TRUE
2. "if $2+2=4$ then $2+3=5$ "
TRUE
3. "if $2+2=5$ then Graham Smith
is the Pope"
TRUE

Let's prove that proposition 3 is true.

Proof

Suppose $2+2 = 5$.

Then $2+2-3 = 5-3$.

Then $1 = 2$.

Graham Ellis and the Pope are
two people.

But $2 \neq 1$, so these two people are
in fact one person.

Therefore Graham Ellis is the Pope.

QED

Truth table for "if p then q "

p	q	if p then q
T	T	T
T	F	F
F	T	T
F	F	T

Notation / terminology

we write

$$p \rightarrow q$$

to denote

if p then q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

we often say " p implies q "
instead of "if p then q ".

Back to the puzzle. Set

p : "A is truthful"

q : "B is truthful"

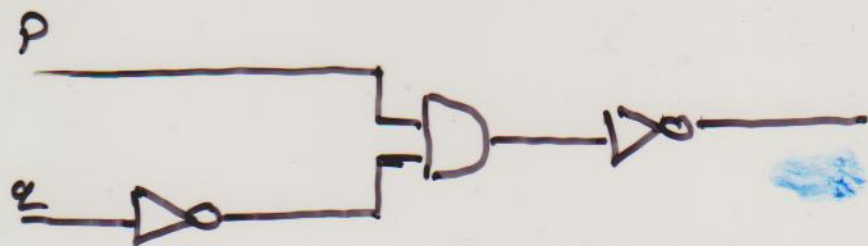
P	Q	A's statement: "if B is truthful then so am I"	B's statement "at least one of us is lying"
T	T	T	F
T	F	T	T
F	T	F	T
F	F	T	T

Only the third row is consistent.
Hence B is truthful and A is lying.

How could we implement \rightarrow as a logic circuit?

P	Q	$\neg(P \wedge \neg Q)$
T	T	T
T	F	F
F	T	T
F	F	T

So \rightarrow can be implemented as
the following logic circuit:



Puzzle You meet three islanders.

A says "if B is lying then so is C"

B says "C is truthful"

C says "at least one of us is lying"

who is lying?

P	Q	R	B lying C lying	C is truthful	at least one is lying
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	F	T	T
F	F	F	T	F	T