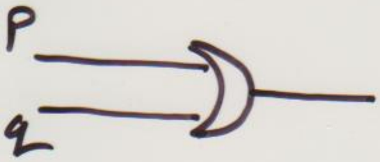
Lecture 3

	p AND q	$p \wedge q$
	p OR q	$p \vee q$
	NOT p	$\neg p$

Translating from English to logic

p: Sue is a first year student

q: Sue is a maths student

1. Sue is a first year maths student

   $p \wedge q$

2. Sue is either a maths student or a first year.

   $p \vee q$

3. Sue is a first year but she is not a maths student.

$$p \wedge (\neg q)$$

4. Sue is neither a maths student nor a first year.

$$(\neg p) \wedge (\neg q)$$

5. Sue is exactly one of the following: a maths student or a first year.

$$(p \vee q) \wedge \neg (p \wedge q)$$

Alternatively 5 could be expressed

$$(p \wedge \neg q) \vee (q \wedge \neg p)$$

Propositional expressions/formula can be analysed using truth tables.

Let's look at the two formulae in 5.

p	q	$p \vee q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

p	q	$p \wedge \neg q$	$q \wedge \neg p$	$(p \wedge \neg q) \vee (q \wedge \neg p)$
T	T	F	F	F
T	F	T	F	T
F	T	F	T	T
F	F	F	F	F

<u>Definition</u> Two propositional formulae are <u>logically equivalent</u> if they have the same truth table.

<u>Theorem</u> (De Morgan's Laws)

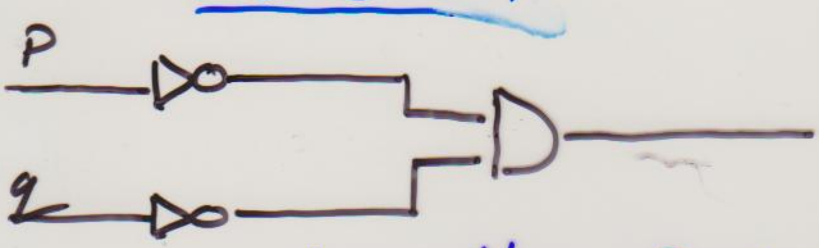
i) $\neg(p \vee q)$ is logically equivalent to $(\neg p \wedge \neg q)$

ii) $\neg(p \wedge q)$ is logically equivalent to $(\neg p \vee \neg q)$

Proof Exercise for you (using truth tables).

Another way of looking at De Morgan's Laws:

Law (i)



has the same effect as

Problem Write the negation of the phrase

"Sue $\overbrace{\text{is a first year}}^{p}$ but she is not $\underbrace{\text{a maths student}}_{q}$"

As a logical expression this statement is

$$p \wedge \neg q$$

So the negation of the statement is

$$\neg (p \wedge \neg q)$$

and this is logically equivalent to

$$\neg p \vee \neg(\neg q)$$

which in turn is equivalent to

$$\neg p \vee q.$$

"Sue is not a first year or she is a maths student"