<u>Proposition</u> For any integer $n \geq 1$ we have

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

<u>Proof</u>
We'll use induction. Let $P(n)$ denote the proposition.

$P(1)$: $1^2 = \frac{1(1+1)(2+1)}{6}$ True

Just now need to show that

$P(n)$ implies $P(n+1)$.

Suppose, for some value of n, that $P(n)$ is true.

so

$$1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Thus

$$1^2+2^2+\cdots+n^2+(n+1)^2 = \frac{n(n+1)(2n+1)}{6}+(n+1)^2$$

$$= \frac{\cdots\cdots}{6}$$

$$= \frac{(n+1)(n+2)(2(n+1)+1)}{6}$$

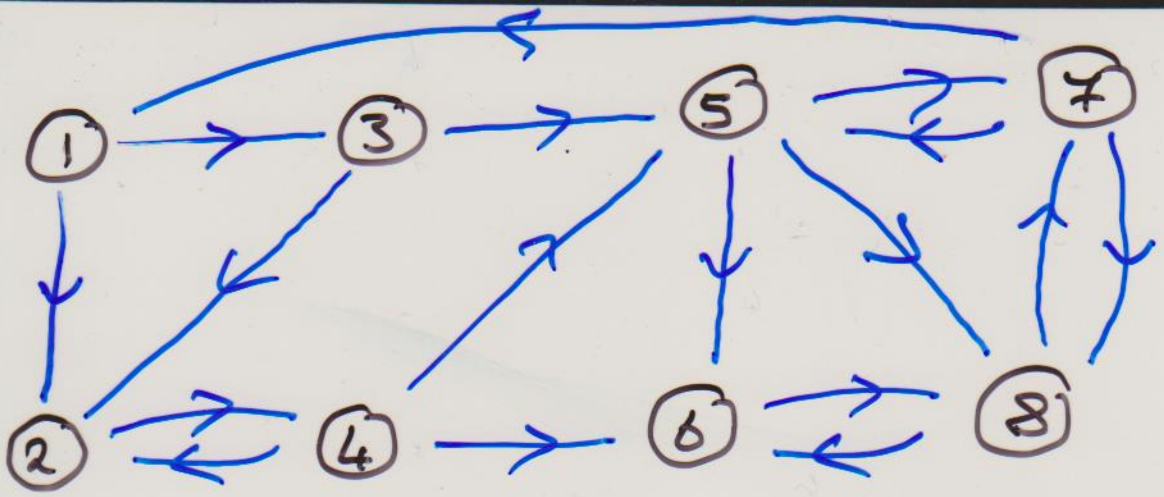
Google

Choose list of key or keywords:

eigenvalue, rabbits, frogs, belly-button, Fibonacci

Those www containing the key words can be represented by a diagram of nodes (one node for each page) and arrows (corresponding to a link from one page to another.

When listing pages a search engine first assigns a number I_n to each page P_n. I_n is the "importance" of P_n.



$$I_1 = \frac{I_7}{3}$$

$$I_2 = \frac{I_1}{2} + \frac{I_3}{2} + \frac{I_4}{3}$$

$$I_3 = \frac{I_1}{2}$$

$$I_4 = I_2$$

$$I_5 = \frac{I_3}{2} + \frac{I_4}{3} + \frac{I_7}{3}$$

$$I_6 = \frac{I_4}{3} + \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_7 = \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_8 = I_6 + \frac{I_5}{3} + \frac{I_7}{3}$$

These equations can be written in matrix notation.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix}$$

eigenvector with eigenvalue equal to 1.

An eigenvector for this example can be computed (more about this later) and is

$$I = \begin{pmatrix} 0.0600 \\ 0.0675 \\ 0.0500 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{pmatrix}$$

Pages will be listed as

P_8
P_6
P_7
P_5
P_2
P_4
P_1
P_3