

## Homework Help

Find  $\gcd(30, 50)$

A)  $50 = 1 \cdot 30 + 20$  Method 1

B)  $30 = 1 \cdot 20 + 10$

C)  $20 = 2 \cdot 10 + 0$

$\gcd(30, 50) = 10$

Alternatively

### Method 2

D)  $30 = \underline{2} \cdot \underline{3} \cdot \underline{5}$

$\gcd(30, 50) = 2 \cdot 5 = 10$

E)  $50 = \underline{2} \cdot \underline{5} \cdot 5$

### Polynomials

Let  $f(x), g(x) \in \mathbb{Q}[x]$ . Their

greatest common divisor is a

polynomial  $h(x)$  where:

$$f(x) = q(x) h(x)$$

$$q'(x), q(x) \in \mathbb{Q}[x]$$

$$g(x) = q'(x) h(x)$$

and  $h(x)$  has highest possible degree.

Problem Find

$$\gcd(x^3 + 2x^2 - 1, x^3 + x^2 + x + 1)$$

in  $\mathbb{Q}[x]$ .

Let's follow method 1.

$$\begin{array}{r} 1 \\ x^3 + 2x^2 - 1 \overline{) x^3 + x^2 + x + 1} \\ \underline{x^3 + 2x^2 - 1} \phantom{+ 1} \\ -x^2 + x + 2 \end{array}$$

A) 
$$x^3 + x^2 + x + 1 = 1(x^3 + 2x^2 - 1) + (-x^2 + x + 2)$$

$$\begin{array}{r} -x - 3 \\ -x^2 + x + 2 \overline{) x^3 + 2x^2 - 1} \\ \underline{x^3 - x^2 - 2x} \phantom{- 1} \\ 3x^2 + 2x - 1 \\ \underline{3x^2 - 3x - 6} \\ 5x + 5 \end{array}$$

B) 
$$x^3 - 2x^2 - 1 = (-x - 3)(-x^2 + x + 2) + 5x + 5$$

$$\begin{array}{r}
 -\frac{1}{5}x + \frac{2}{5} \\
 5x+5 \overline{) -x^2+x+2} \\
 \underline{-x^2-x} \phantom{+2} \\
 2x+2 \\
 \underline{2x+2} \\
 0
 \end{array}$$

$$c) -x^2+x+2 = \left(-\frac{1}{5}x + \frac{2}{5}\right)(5x+5) + 0$$

Thus

$$\gcd(x^3+2x^2-1, x^3+x^2+1) = 5x+5.$$

Also  $k(5x+5)$  is a gcd for

any  $0 \neq k \in \mathbb{Q}$ . So, for

instance  $\frac{1}{5}(5x+5) = x+1$  is

a gcd.

Now let's re-do the problem using method 2.

$$f(x) = x^3 + 2x^2 - 1$$

$f(-1) = 0$ , so  $(x+1)$  divides  $f(x)$ .



$$f(x) = x^3 + 2x^2 - 1 = (x+1)(x^2 + x - 1)$$

$$g(x) = x^3 + x^2 + x + 1$$

$g(-1) = 0$ , so  $(x+1)$  divides  $g(x)$ .

$$g(x) = x^3 + x^2 + x + 1 = (x+1)(x^2 + 1)$$

↑  
irreducible

Hence  $\gcd = x+1$ .

Last homework question,

TRUE/FALSE

- Mathematical induction is used to prove statements about all real numbers. **FALSE**
- Every proof by Mathematical induction requires at least two base cases to be proved. **FALSE**
- Mathematical induction differs from the kind of induction used in experimental science as it is actually a form of deductive reasoning. **TRUE**