# Algebra MA1064/160

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- Logic ~ computer arithmetic
- Polynomials ~ digit communications
- (More) matrix algebra ~ Google page rank

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## Logic

$5 + 6 = 11$

Decimal arithmetic

$2012 = 2.10^3 + 0.10^2 + 1.10 + 2$

Binary arithmetic

$5 = 1.2^2 + 0.2 + 1$

So 101 is the binary representation of 5.

$2012 = 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2$

11111011100 is the binary representation of 2012.

$6 = 2^2 + 2 + 0$

110 represents 6.

Let's add 5+6 using the binary system.

$$\begin{array}{r} 101 \\ 110 \\ \hline 1011 \end{array}$$

So

101+110 = 1011

How is this related to logic?

| I am Walsh | I like maths | I am Walsh AND I like maths |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

A basic adding operation used in the addition of binary numbers is:

| A | B | Sum | Carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |

↑ similar to AND

Back to logic

| I am Welsh | I like maths | I am Welsh OR I like maths BUT NOT BOTH |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

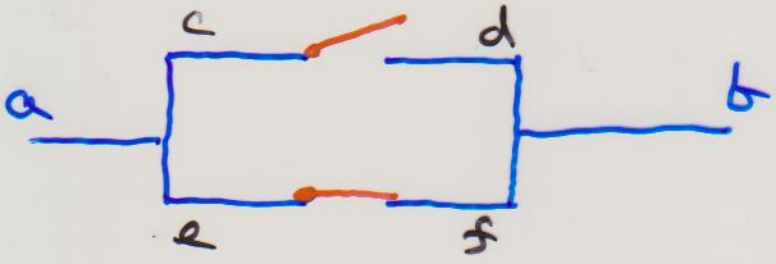
↑ similar to sum

Now let's consider some electric relay switches


ab is Open : 0


ab is Closed : 1

with this notation :




| cd | ef | ab |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |


| ab | ac |
|---|---|
| 0 | 1 |
| 1 | 0 |

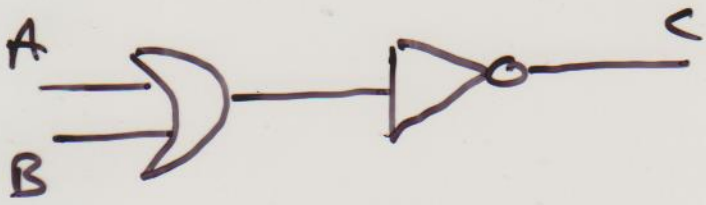
## Some symbols & names


| A | B |
|---|---|
| 0 | 1 |
| 1 | 0 |

NOT


| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

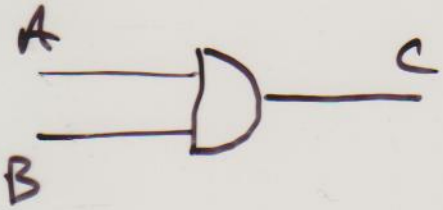
Combining:




| A | B | C |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

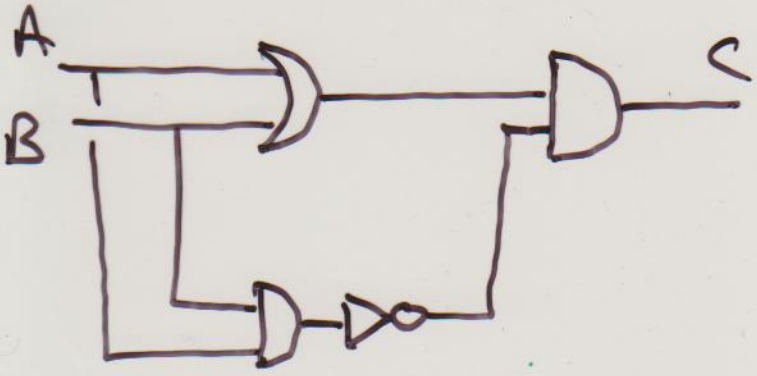
NOR

often written as




| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

AND




| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

XOR