NUI Galway
OÉ Gaillimh

Semester I Examinations 2011

Exam Code	1BME1
Exam	First Year Arts (Mathematics and Education)
Module	MATHEMATICS
Module Code	MA185
Paper No	1
External Examiner(s)	Dr. C. Campbell
Internal Examiner(s)	Dr. J. Aramayona Prof. G. Ellis
Instructions	Answer any FOUR questions. You cannot score more than 44% in this exam unless you score at least 60% on one question in each section.
Duration	2 Hours
No. of Pages	4 Pages (including this cover page)

Requirements:

Release to Library:	Yes
Other Materials	Non-programmable calculators

CALCULUS SECTION

1. (a) Calculate the following limits:

$$\lim_{x \to -4} \frac{x^2 + x - 12}{x^2 + 2x - 8} \qquad \text{and} \qquad \lim_{x \to +\infty} \left(\sqrt{x^2+1} - \sqrt{x^2-1} \right).$$

(b) Find all the possible real numbers a, b for which the function

$$f(x) = \begin{cases} x^2 + ax + 1, & \text{if } x < 2 \\ x^3 + 1, & \text{if } 2 \leq x < 3 \\ 4x^2 + b, & \text{if } x \geq 3 \end{cases}$$

is continuous on $\mathbb{R}$.

2. (a) Prove that the equation

$$x^3 + 3x + 11 = 0$$

has exactly one real root.

(b) Consider the function

$$f(x) = x^4 - 2x.$$

(i) Find all critical points of f. For each critical point, decide whether it is a maximum, a minimum, or neither.

(ii) Find the intervals on which f increases/decreases.

3. (a) Find an antiderivative of the function

$$f(t) = e^{3t}.$$

(b) The number $N(t)$ of tumor cells at time t in a given tissue is described by the differential equation

$$\frac{dN}{dt} = e^{3t},$$

where t is measured in days. Supposing that $N(0) = 1000$, calculate the number of tumoral cells after 4 days.

(c) Solve

$$e^{x^2-1} = 7 \ .$$

ALGEBRA SECTION

4. (a) Decipher:

$$TMY$$

This short ciphertext was produced by applying the enciphering function $f_E\colon \mathbb{Z}_{26} \to Z_{26}, x \mapsto 9x+16$ to single letter message units over the alphabet $A = 0, B = 1, ...Z = 25$.

(b) Calculate the number $\phi(180)$ of integers from 1 to 180 that are coprime to 180. Then calculate

$$7^{98} \mod 180.$$

(c) A basket contains x eggs. Counting in twos there is one egg left over, counting in threes there are 2 eggs left over, and counting in fives there is 1 egg left over. What is the smallest possible value for x?

5. (a) The ciphertext

$$ESDCWNMH$$

was produced by applying the function

$$f_E\colon \begin{pmatrix} x \\ y \end{pmatrix} \mapsto A \begin{pmatrix} x \\ y \end{pmatrix} \qquad \text{with } A = \begin{pmatrix} 2 & 3 \\ 7 & 5 \end{pmatrix}$$

to 2-letter message units over the alphabet $A = 0, B = 1, ..., Z = 25$. Calculate A^{-1} mod 26 and hence determine the first FOUR letters of plaintext.

(b) Use row operations to find the inverse of

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}.$$

6. (a) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in the line $y = x$ and let $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ be clockwise rotation through 90^o about the origin. Find the point $v = (x, y) \in \mathbb{R}^2$ such that $g(f(v)) = (3, 4)$.

(b) Consider

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \phi = \frac{1+\sqrt{5}}{2}, \quad v_1 = \begin{pmatrix} \phi \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ -\phi \end{pmatrix}.$$

(i) Determine the characteristic polynomial $p_A(\lambda)$ of A.

(ii) Verify that v_1 and v_2 are eigenvectors of A and determine the corresponding eigenvalues.

(iii) Find a diagonal matrix D and invertible matrix T such that $A = TDT^{-1}$.