Computational homology in dynamical systems

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29 June - 10 July 2009

Outline

- Dynamical systems
- Rigorous numerics of dynamical systems
- Homological invariants of dynamical systems
- Computing homological invariants
- Homology algorithms for subsets of $\mathbb{R}^d$
- Homology algorithms for maps of subsets of $\mathbb{R}^d$
- **Applications**

Applications [3]

- **Lorenz equations**:
  K. Mischaikow, A. Szymczak, MM, *J. Diff. Equ.*, 2001.
- **Hénon map**:
  S. Day, R. Frongilo, R. Trevino, in preparation.
- **Kot-Schaffer map in** $L^2([-\pi, \pi])$:
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Subshifts of finite type

- For a $k \times k$ matrix $A = (A_{ij})$ over $\mathbb{Z}_2$ put

$$\Sigma(A) := \left\{ \alpha \in \Sigma_k \mid \forall i \in \mathbb{Z} A_{\alpha(i)\alpha(i+1)} = 1 \right\}$$

- $\Sigma(A)$ is invariant under the shift map σ
- $(\Sigma(A), \sigma_{|\Sigma(A)})$ - subshift of finite type

Theorem. $h(\Sigma(A)) = \log(\lambda(A))$

Lorenz equations: classical parameter values

Theorem. (K. Mischaikow, A. Szymczak, MM, 2001) Consider the Lorenz equations and the plane $P := \{(x, y, z) \mid z = 27\}$. For all parameter values in a sufficiently small neighborhood of $(\sigma, R, b) = (28, 10, 8/3)$ there exists a Poincaré section $N \subset P$ such that the associated Poincaré map is Lipschitz and well defined. Furthermore, for

$$A := \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

there is a continuous surjection $\rho : \mathrm{Inv}(N, g) \to \Sigma(A)$ such that

$$\rho \circ g = \sigma \circ \rho.$$

In particular $h(\mathrm{Inv}(N, g)) \geq 0.48$ Moreover, for every $\alpha \in \Sigma(A)$ which is periodic there exists an $x \in \mathrm{Inv}(N, g)$ on a periodic trajectory such that $\rho(x) = \alpha$.

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Hénon map

Consider the Hénon map $h : \mathbb{R}^2 \to \mathbb{R}^2$ given by the formula

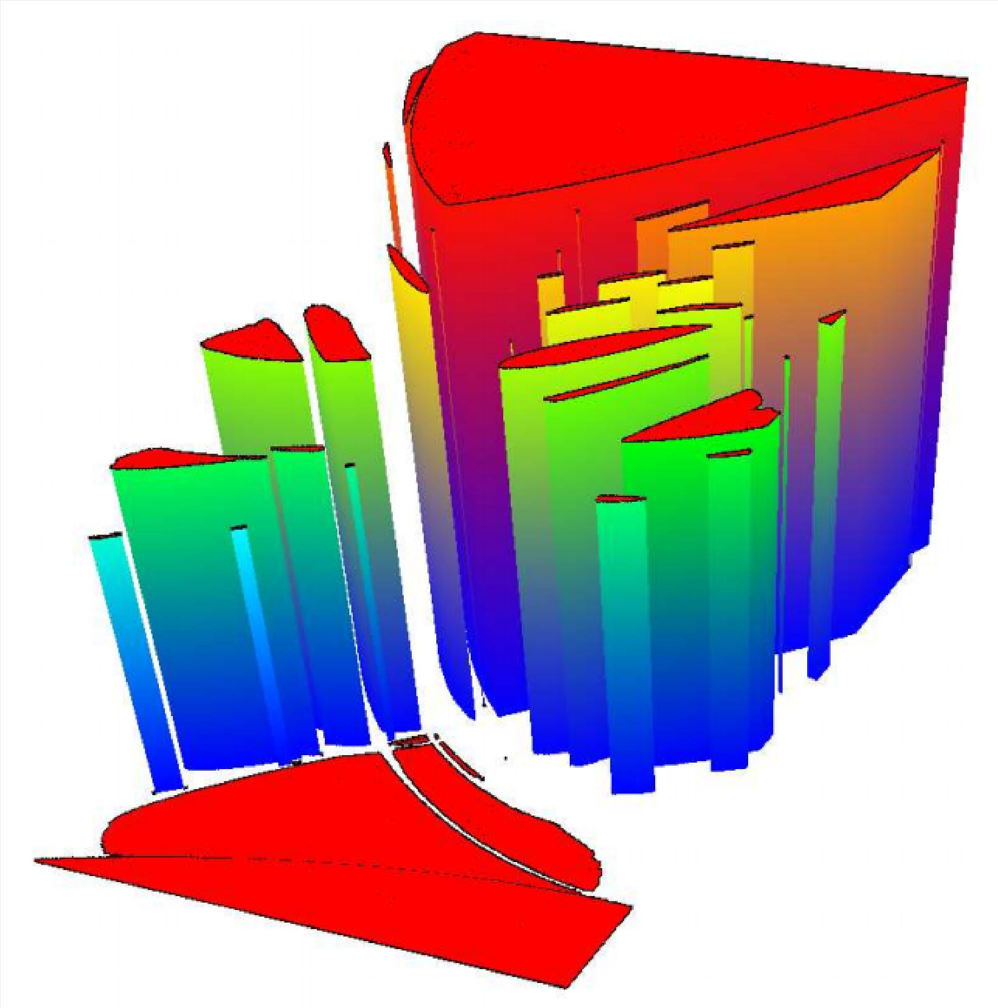
$$h(x, y) := (1 + y/5 - ax^2, 5bx)$$

at the classical parameter values $a = 1.4$ and $b = 0.2$.

Theorem. (T. Kaczynski, K. Mischaikow, MM, 2004) Let S be the maximal invariant subset of the union of constructed connected components C_j. Then there exists a semiconjugacy with a subshift of finite type on 8 symbols and topological entropy $h = 0.28$ such that for each periodic sequence $\theta \in \Sigma_A$ with period p $\rho^{-1}(\theta)$ contains a periodic orbit with period p. In particular $h(S) \geq 0.28$.

Theorem. (S. Day, R. Frongilo, R. Trevino, 2009) The classical Hénon map admits an isolated invariant set S semi-conjugated to a subshift on 129 symbols and $h(S) \geq 0.42$.

Hénon map



Hénon plateaus (Z. Arai, 2007) with lower entropy bounds (R. Frongilo, 2009)

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Kot-Schaffer growth-dispersal model for plants

$$\Phi : L^2([-\pi,\pi]) \to \Phi : L^2([-\pi,\pi])$$

is given by

$$\Phi(a)(y) := \frac{1}{2\pi}\int_{-\pi}^{\pi} b(x,y)\ \ \mu\, a(x)\left(1 - \frac{a(x)}{c(x)}\right) dx,$$

where $\mu > 0$.

- Via Fourier expansion the map is replaced by a map acting on an infinite sequence of Fourier coefficients
- By the method of the self-consistent apriori bounds (P. Zgliczyński, K. Mischaikow, 2001) the study of dynamics is reduced to a discrete dynamical system in $\mathbb{R}^1 1$.

Infinite dimensional dynamics

k	$(p^*)_k$
0	1.01701222469896
1	0.194337336695483
2	0.0305189885313998
3	0.00388416812157288
4	0.000406689870061427
5	$3.59686642677538 \cdot 10^{-5}$
6	$2.75312512992254 \cdot 10^{-6}$
7	$1.85783274541004 \cdot 10^{-7}$
8	$1.12063486092261 \cdot 10^{-8}$
9	$6.10776632026745 \cdot 10^{-10}$
10	$3.03735627584813 \cdot 10^{-11}$

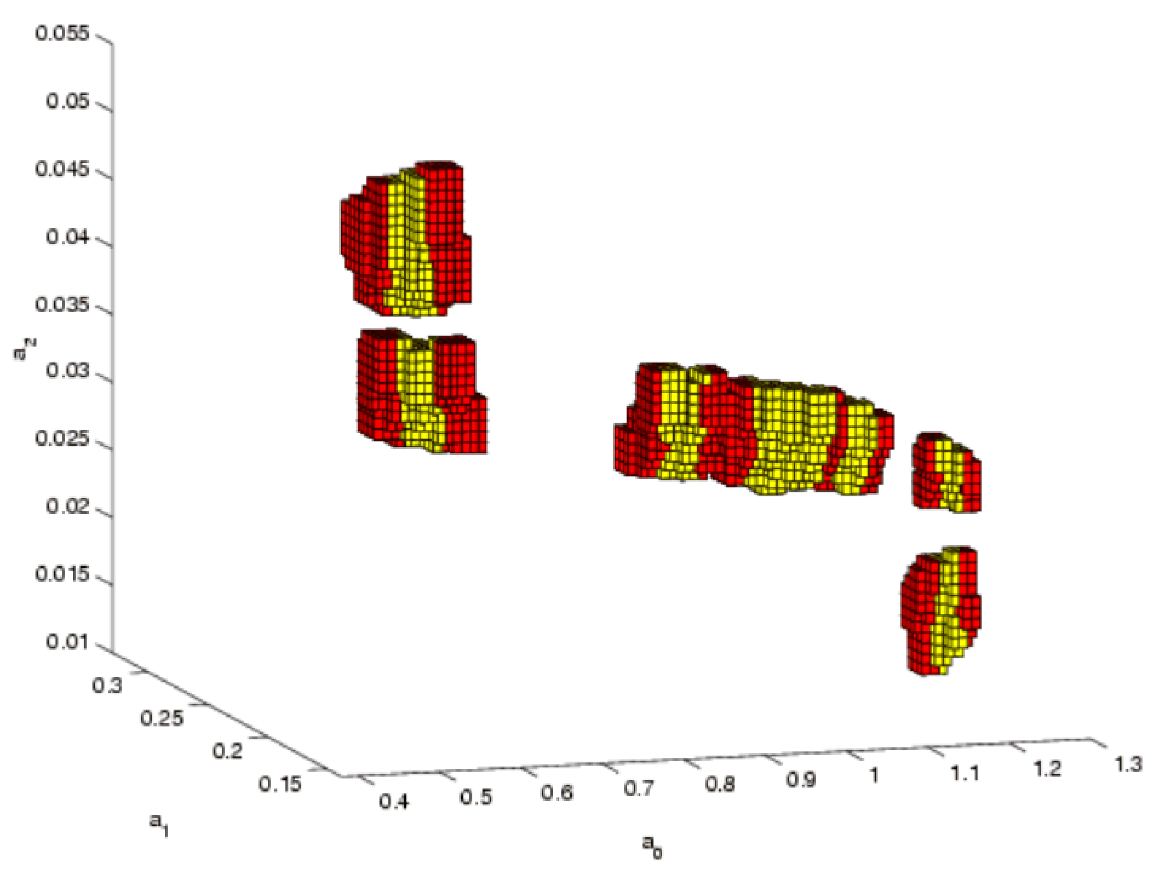
TABLE 6.3
First 11 Fourier coefficients of p^*.

k	$(q_1^*)_k$	$(q_2^*)_k$
0	0.612404314978377	1.20068110795964
1	0.240548407216622	0.191505924758062
2	0.0427803169880592	0.0186218996298865
3	0.00371012084664812	0.00293452057215404
4	0.000286872331997756	0.000493085588431086
5	$2.85759422150114 \cdot 10^{-5}$	$4.83837782923445 \cdot 10^{-5}$
6	$2.46114548652796 \cdot 10^{-6}$	$3.16438225821576 \cdot 10^{-6}$
7	$1.61009990418761 \cdot 10^{-7}$	$1.78892095960431 \cdot 10^{-7}$
8	$9.62062994427963 \cdot 10^{-9}$	$1.00726290466515 \cdot 10^{-8}$
9	$5.95120614532175 \cdot 10^{-10}$	$5.28784988682338 \cdot 10^{-10}$
10	$3.43628779469884 \cdot 10^{-11}$	$2.46555443216212 \cdot 10^{-11}$

TABLE 6.4
First 11 Fourier coefficients of q_1^* and q_2^*.

Theorem. Kot-Schaffer system admits a stationary solution p and a 2-periodic solution q in the ball of radius 10^{-11} in L^2 and C^0 norms respectively around the functions p^* and q^* whose non-zero Fourier coefficients are given above. Moreover, there exists a heteroclinic connection running from a neighborhood of p to a neighborhood of q.

Infinite dimensional dynamics



Projection onto $\mathbb{R}^3$ of one of the index pairs used in the proof.

Theorem. Kot-Schaffer system admits an invariant set with entropy not less than 0.18

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- X — a compact submanifold of $\mathbb{R}^{d_0}$
- $f : X \to X$ — a physical process governed by a smooth discrete dynamical system with unknown f
- $\mu : X \to \mathbb{R}$ — a smooth form on X accessible by measurement
- for $d > 2d_0$ define

$$\mu_d : X \ni x \to (\mu(x), \mu(f(x)), \ldots, \mu(f^{d-1}(x))) \in \mathbb{R}^d$$

A time series of f is a sequence $\tau = \{\tau_i\}_{i=1}^n \subset \mathbb{R}^n$ with large n such that $\tau_i = \mu(f^i(x))$ for some $x \in X$.

- $\mathcal{K}_\delta^d := \delta\mathcal{K}^d$ — a δ-rescalling of $\mathcal{K}^d$ for some $\delta > 0$
- $\tau_i^d := (\tau_i, \tau_{i+1}, \ldots, \tau_{i+d-1})$
- $\Theta_d^\tau := \{\, \tau_i^d \mid 1 \le i \le n-d+1 \,\}$
- $\mathcal{X}^\tau := \{\, Q \in \mathcal{K}_\delta^d \mid Q \cap \Theta_d^\tau \ne \emptyset \,\}$
- $\mathcal{F} : \mathcal{X}^\tau \rightrightarrows \mathcal{X}^\tau$ given by

$$Q \to \{\, P \in \mathcal{X}^\tau \mid \tau_i^d \in Q \text{ and } \tau_{i+1}^d \in P \text{ for some } 1 \le i \le n-d \,\}$$

$B \subset \mathbb{R}^d$ is δ-stiff if for every $\mathcal{C} \subset \mathcal{K}^d_\delta$ the set $|\mathcal{C}| \cap B$ is a deformation retract of $|\mathcal{C}|$.

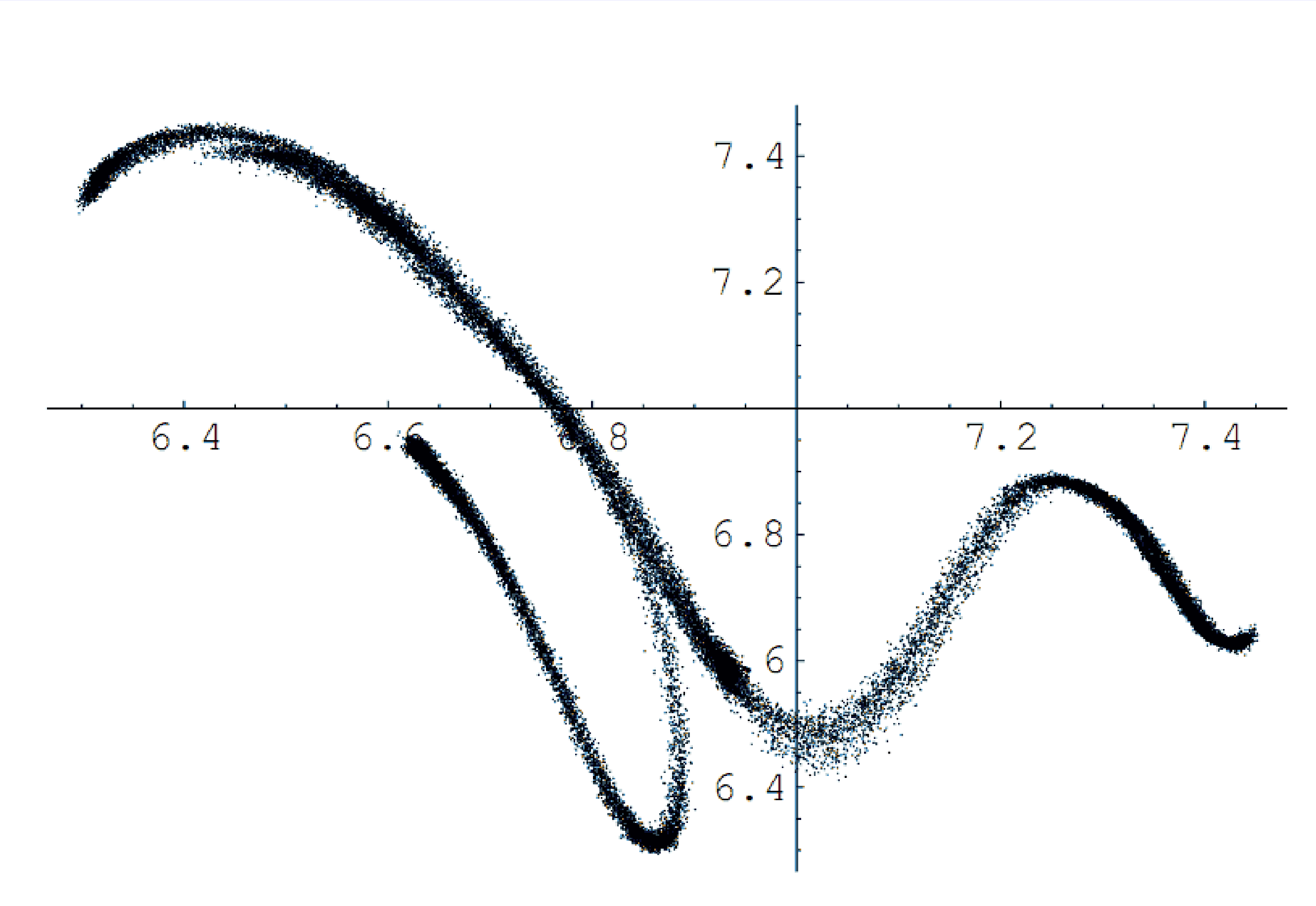
Theorem. Assume $\mu^d(X)$ is δ-stiff, cF is acyclic valued with a continuous selector and $\mathcal{N} \subset \mathcal{K}^d_\delta$ is an isolating neighborhood for $\mathcal{F}$. Then

$$\mathrm{Con}(\mathcal{N}, \mathcal{F}) = \mathrm{Con}((\mu^n)^{-1}(|\mathcal{N}|), f).$$

Time series analysis of experiments

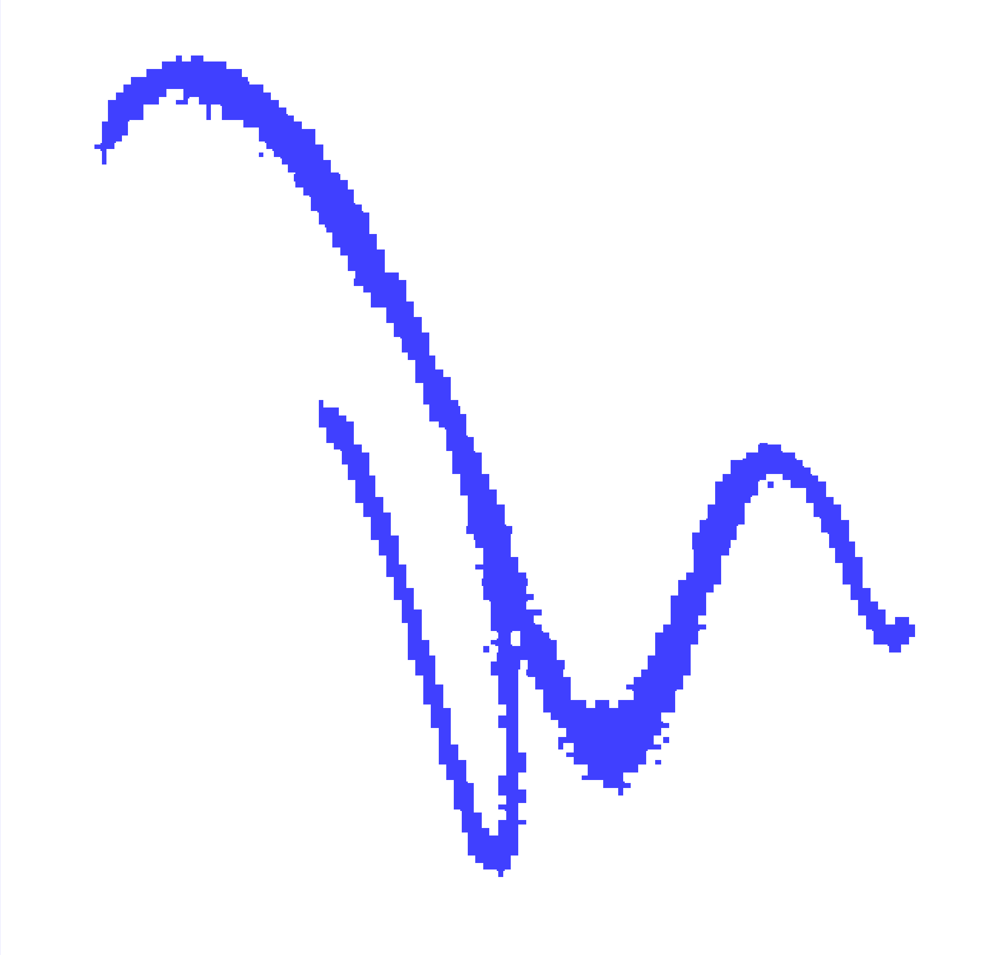
- magnetoelastic ribbon — a thin strip of material with varying modulus of elasticity under magnetic field
- a ribbon clamped from the bottom buckles under its own weight
- experiment: the motion of the ribbon was measured by means of a photonic sensor
- A typical outcome: 100 000 consecutive data points

Time series analysis of experiments



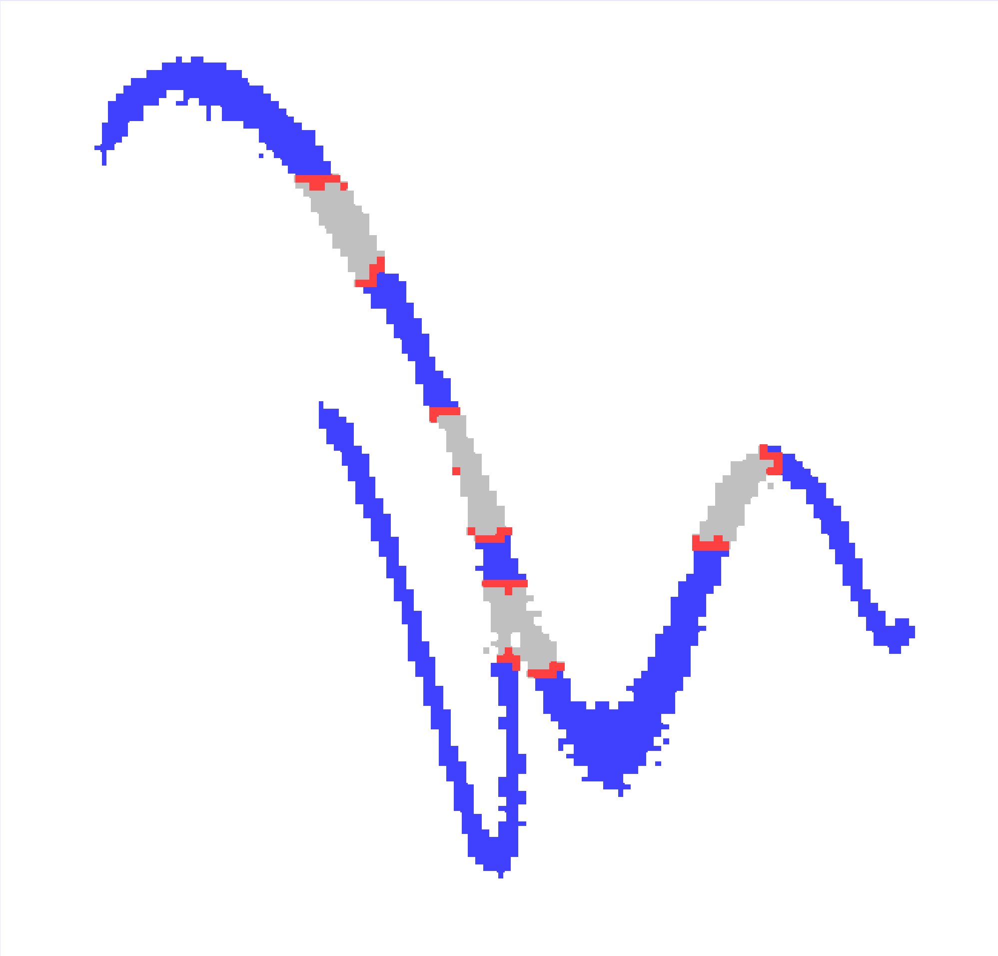
Plot of the points for $d = 2$.

Time series analysis of experiments



The covering $\mathcal{N} \subset \mathcal{K}_\delta^2$ of $\mu^d(X)$ for $\delta = 0.0106$

Time series analysis of experiments



The isolating neighborhood N with and index pair $P_1 = N$ (gray and red) and P_2 (red)

The resulting index map has the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

This leads to the conclusion that the topological entropy of the dynamics of the ribbon must be at least 0.38.

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Parametrized dynamical systems

- X — locally compact subset of $\mathbb{R}^d$
- Λ — compact, locally contractible, connected subset of $\mathbb{R}^q$
- a parametrized family of discrete semidynamical systems:

$$f\colon X \times \Lambda \ni (x, \lambda) \mapsto f(x, \lambda) = f_\lambda(x) \in X$$

- problem: determine regions of parameter λ where the global asymptotic dynamics is common
- importance: biology, social sciences
- unified semidynamical system $F\colon X \times \Lambda \to X \times \Lambda$

$$F(x, \lambda) = \big(f_\lambda(x), \lambda\big) = \big(f(x, \lambda), \lambda\big).$$

- $A_{\Lambda_0} := A \cap X \times \Lambda_0$ for $A \subset X \times \Lambda$ and $\Lambda_0 \subset \Lambda$
- $F_{\Lambda_0} := F_{|\Lambda_0}$ for $\Lambda_0 \subset \Lambda$

Morse decompositions

- S — an isolated invariant set of F

A Morse decomposition of S is a finite collection

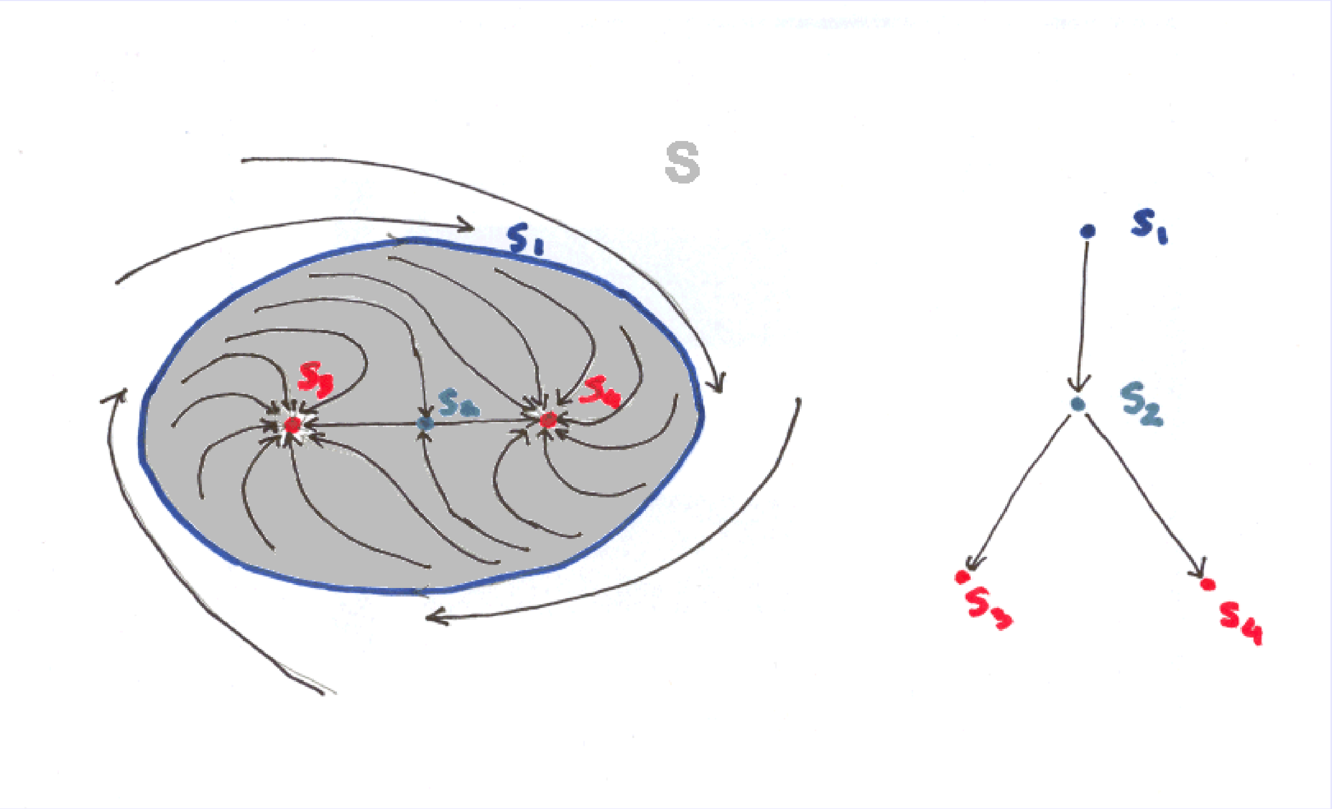
$$\mathbf{M}(S) = \{\, M(p) \subset S \mid p \in \mathcal{P} \,\}$$

of disjoint isolated invariant sets of F, called Morse sets, which are indexed by the set $\mathcal{P}$ on which there exists a partial order $>$, called an admissible order, such that for every $(x, \lambda) \in S \setminus \bigcup_{p \in \mathcal{P}} M(p)$ and any trajectory γ of F through (x, λ) in S there exist indices $p > q$ such that under F

$$\omega(\gamma) \subset M(q) \quad \text{and} \quad \alpha(\gamma) \subset M(p).$$

- Morse graph of $\mathbf{M}(S)$ is an oriented graph with vertices in $\mathcal{P}$ and edges given by minimal relations $p > q$
- Conley-Morse graph — a Morse graph labeled at vertices by the Conley indices of the respective isolated invariant sets

Morse decompositions



Morse decomposition and the Morse graph of an isolated invariant set S.

Morse decompositions

- $\mathcal{X} \subset \mathcal{K}_\delta^d$ — a finite subfamily
- $\mathcal{F}$ — a combinatorial enclosure of F
- $\mathcal{S}$ — an invariant set of $\mathcal{F}$

A combinatorial Morse decomposition of $\mathcal{S}$ is a collection

$$\{\,\mathcal{M}(p) \subset S \mid p \in \mathcal{P}\,\}$$

of disjoint minimal invariant sets of $\mathcal{F}$, called combinatorial Morse sets.

Note that there is a natural partial order $>$ on $\mathcal{P}$ given by $p > q$ iff there exists a finite trajectory from an element of $\mathcal{M}(p)$ to an element of $\mathcal{M}(q)$

Morse decompositions

Assumptions:

- $X = |\mathcal{X}|$ for some $\mathcal{X} \subset \mathcal{K}_\delta^d$
- $\Lambda = |\mathcal{Q}|$ for some $\mathcal{Q} \subset \mathcal{K}_\delta^q$
- $S := \mathrm{Inv}(B, F)$ — an isolated invariant set of F for some isolating neighborhood $B \subset X \times \Lambda$
- for each $Q \in \mathcal{Q}$ the set $\mathcal{B}_Q := \mathcal{X}\big(\bigcup_{\lambda \in Q} B_\lambda\big)$ satisfies $S_Q = \mathrm{Inv}\,\big(|\mathcal{B}_Q| \times Q, F_Q\big)$

Theorem. Let $Q \in \mathcal{Q}$ and let $\{\mathcal{M}_Q(p) \mid p \in \mathcal{P}_Q\}$ be the combinatorial Morse decomposition for $\mathcal{F}_Q$. If $\mathcal{F}_Q\big(\mathcal{M}_Q(p)\big) \subset \mathcal{B}_Q$ for all $p \in \mathcal{P}_Q$, then the Morse graph for the combinatorial Morse decomposition is the Morse graph for the Morse decomposition of S_Q defined by

$$\mathbf{M}(S_Q) := \{\, \mathrm{Inv}\,(|\mathcal{M}_Q(p)| \times Q, F_Q) \mid p \in \mathcal{P}_Q \,\}.$$

Moreover, each $|\mathcal{M}_Q(p)|$ is an isolating neighborhood for $\mathrm{Inv}\,|\mathcal{M}_Q(p)|$.

- For $P, Q \in \mathcal{Q}$ such that $P \cap Q \neq \emptyset$ put
$$R := \{ (p, q) \in \mathcal{P}_P \times \mathcal{P}_Q \mid \mathcal{M}_P(p) \cap \mathcal{M}_q(Q) \neq \emptyset \}.$$
- If R is a graph isomorphism, then we say that P and Q share a Morse graph
- The equivalence classes with respect to the transitive closure of this relation are called continuation classes.

Theorem. The Conley-Morse graphs in the same continuation class coincide.

The continuation graph consists of the continuation classes as vertices with edges joining two classes $[P]$ and $[Q]$ iff there exist $P_0 \in [P]$ and $Q_0 \in [Q]$ such that $P_0 \cap Q_0 \neq \emptyset$.

The continuation graph constitutes a database which captures the types of dynamics detectable in the parametrized dynamical system at a given level of resolution.

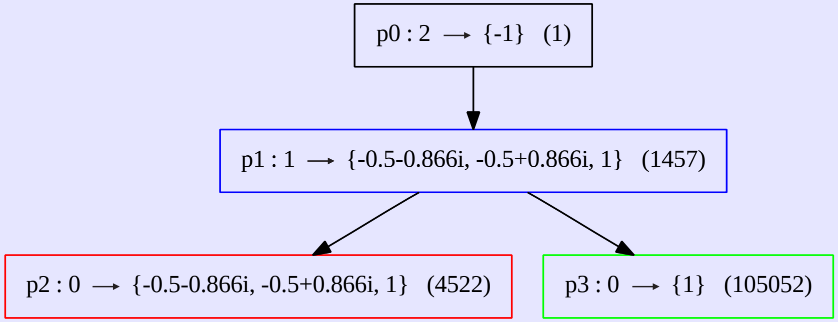
Leslie population model

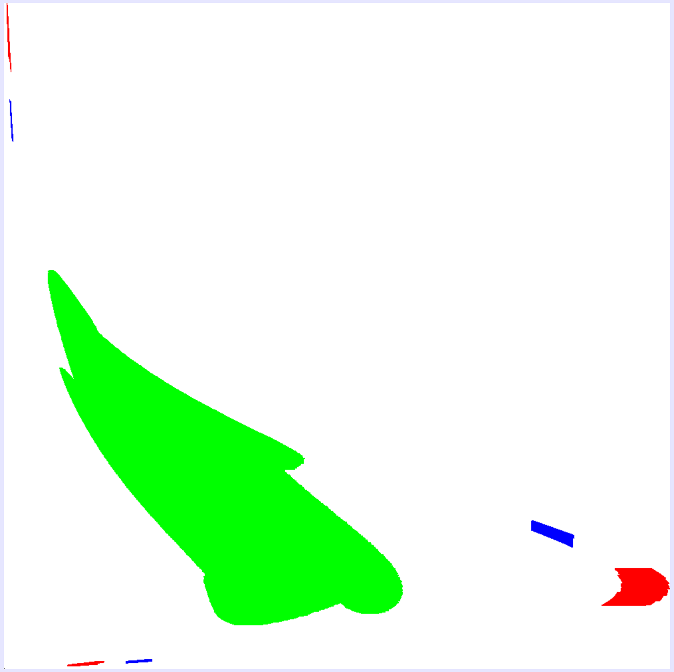
$$(x,\lambda) = \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \theta_2 \\ p \end{bmatrix} \right) \mapsto f(x,\lambda) = \begin{bmatrix} (\theta_1 x_1 + \theta_2 x_2) e^{-0.1(x_1+x_2)} \\ p x_1 \end{bmatrix}$$

Take

$$\begin{aligned} \Lambda &:= \{ (\theta_1, \theta_2) \in [8, 37] \times [3, 50] \}, \\ X &:= [-0.001, 320.056] \times [-0.001, 224.040] \end{aligned}$$

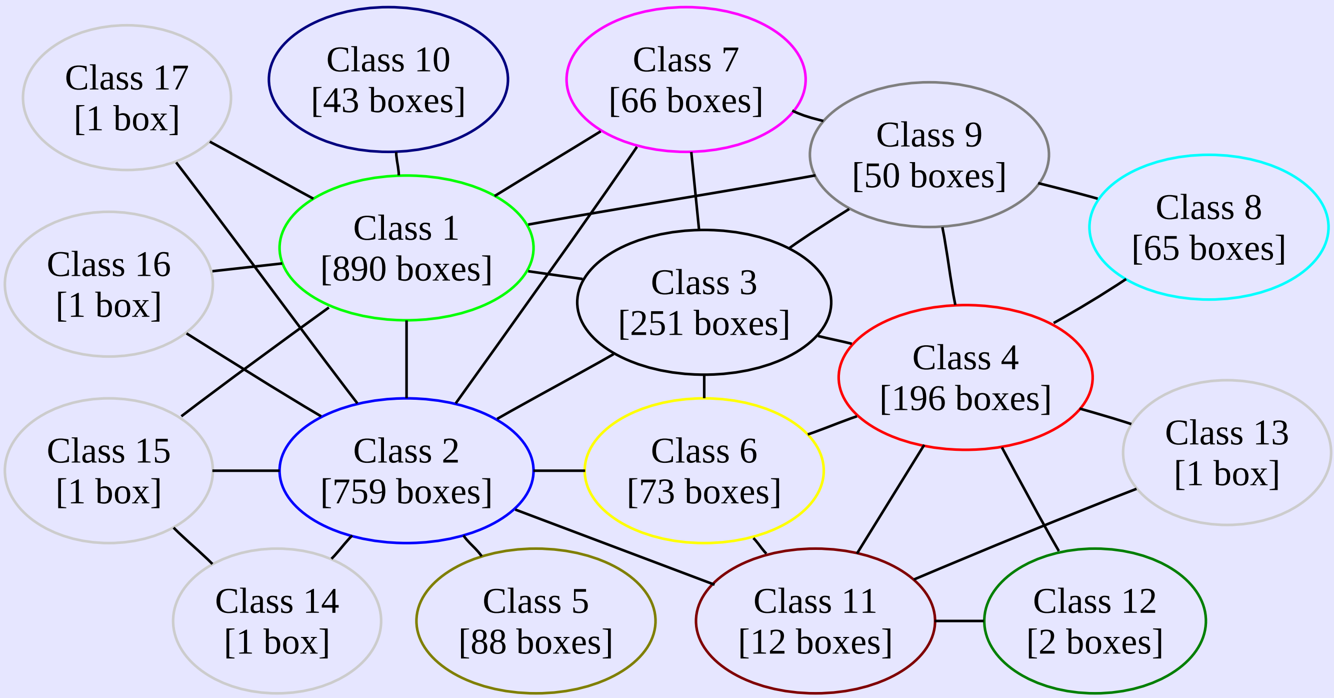
Leslie population model





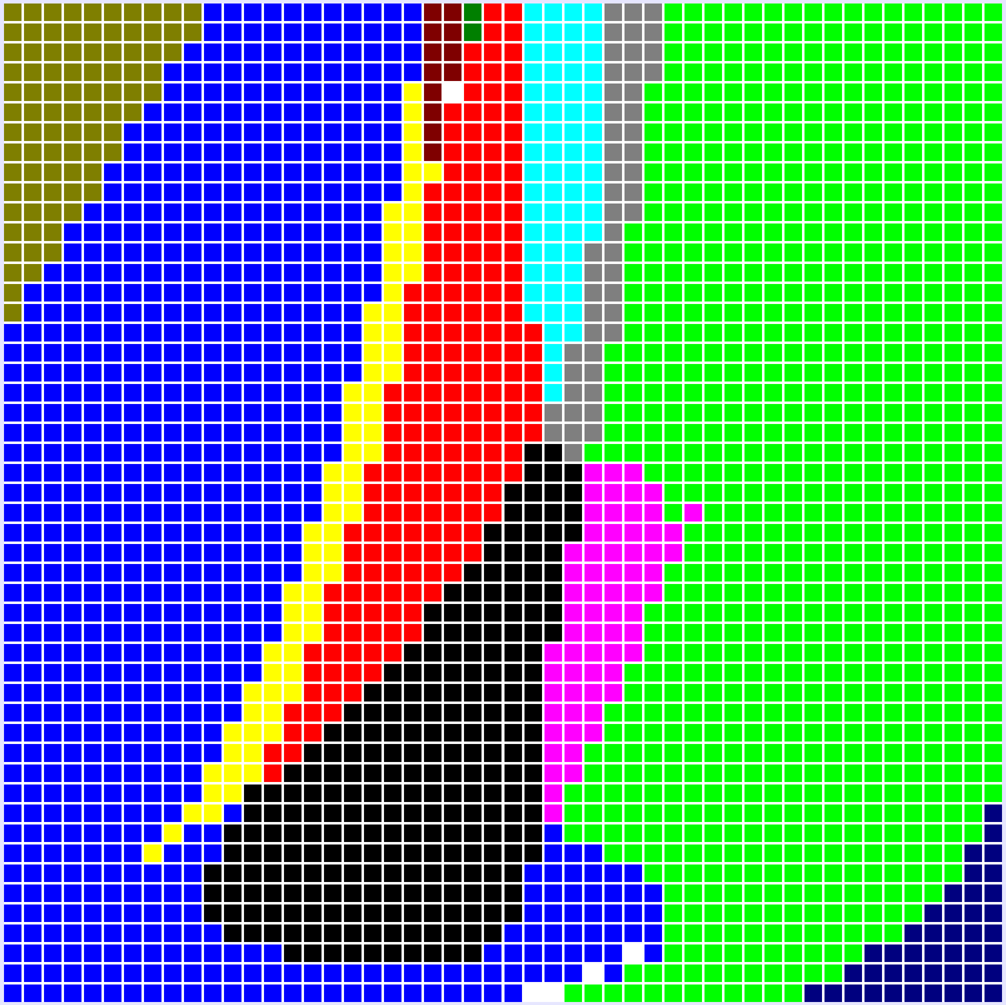
One of the Conley-Morse graphs and the corresponding combinatorial Morse sets.

Leslie population model



The continuation graph.

Leslie population model



The continuation diagram.

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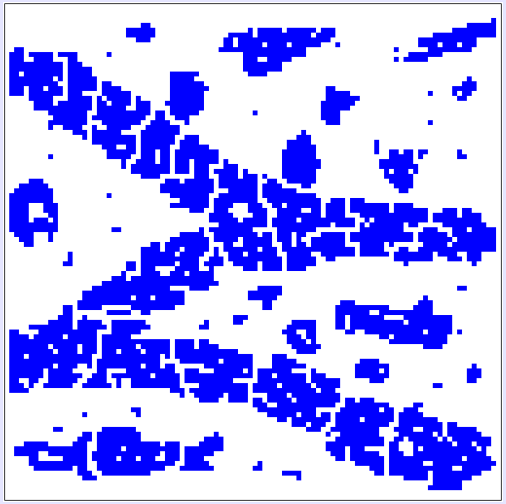
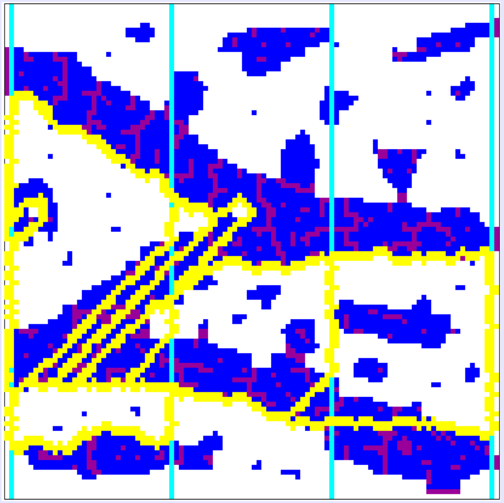
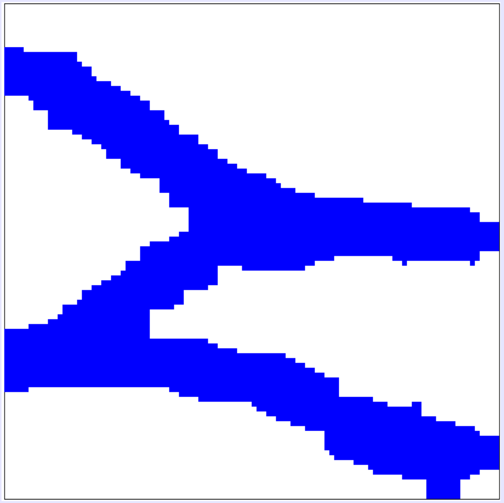
Extracting linear features







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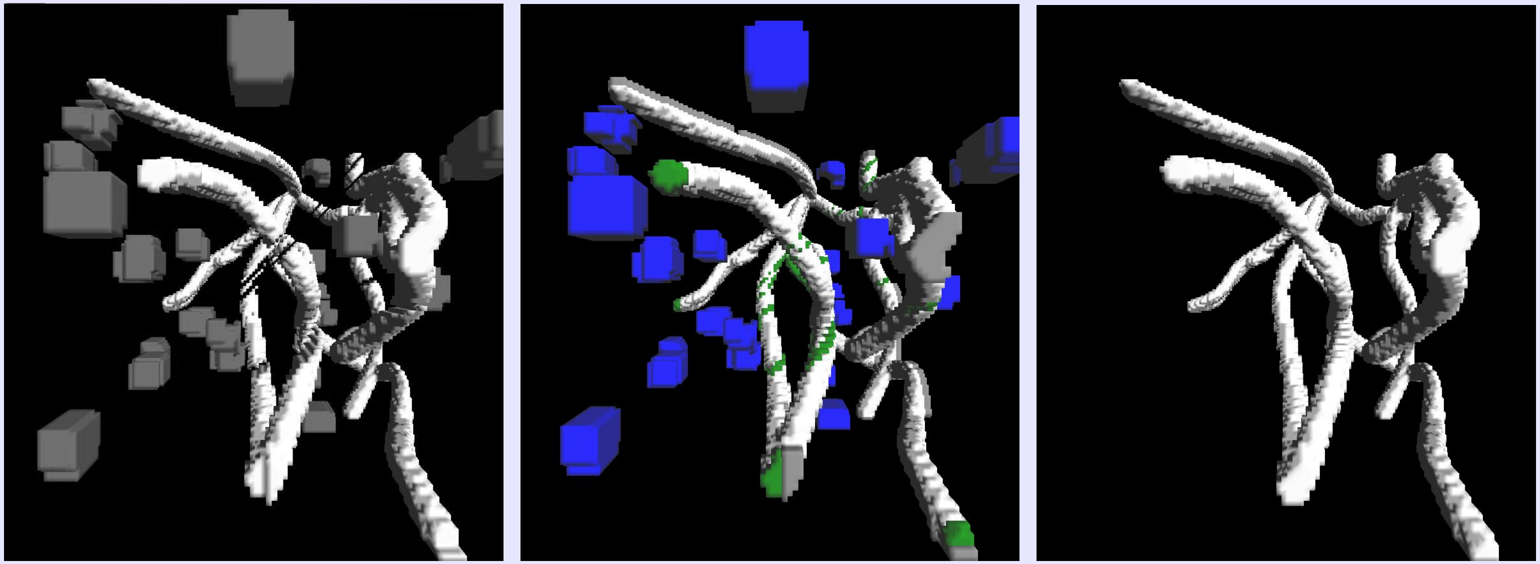

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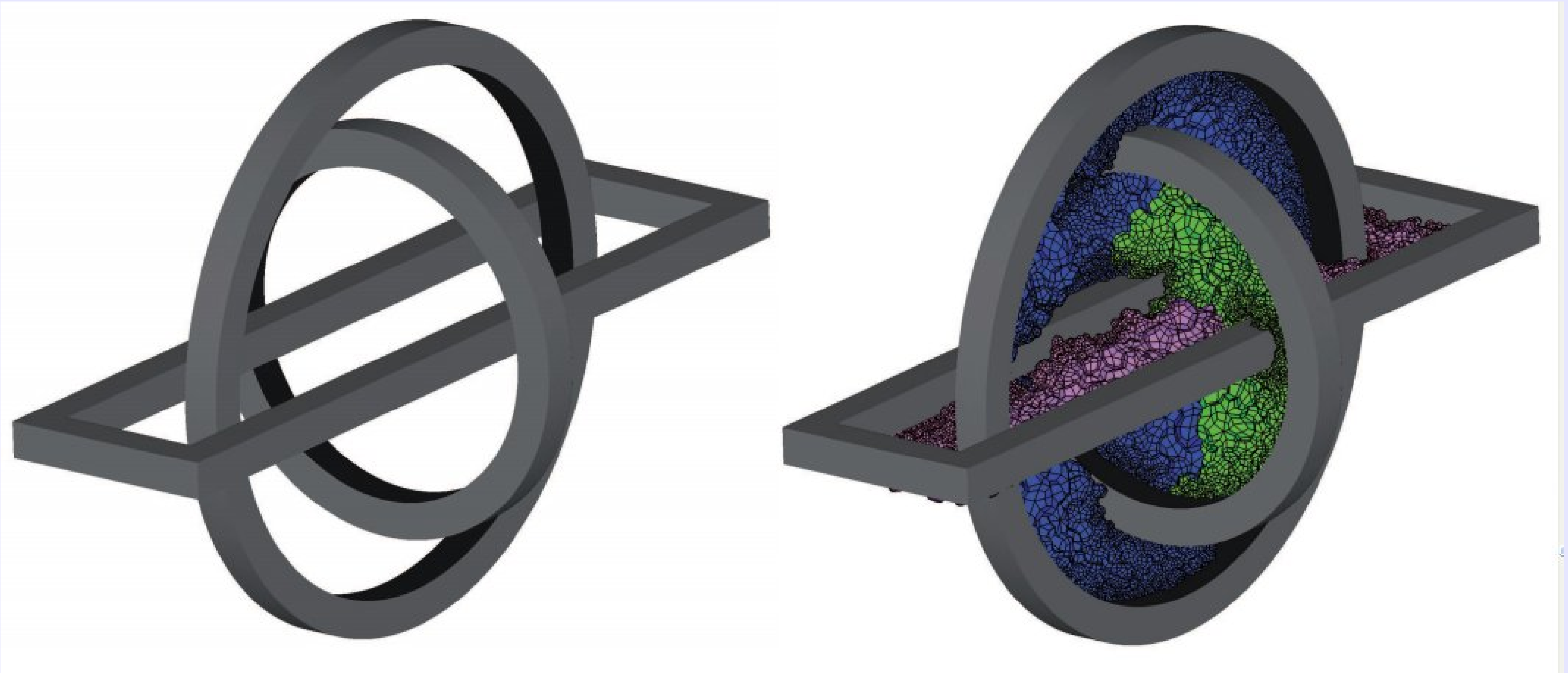
- **Time series dynamics**:
  K. Mischaikow, J. Reiss, A. Szymczak, MM, *Phys. Rev. Lett.*, 1999.
- **Databases for multiparameter systems**:
  Z. Arai, H. Kokubu, W. Kalies, K. Mischaikow, H. Oka, P. Pilarczyk, *SIAM J. App. Dyn. Sys.*, accepted.
- **Image analysis**:
  A. Krajniak, M. Żelawski, *CISP'09*, accepted.
- **Generation of cuts in electromagnetism**:
  P. Dłotko, R. Specogna, F. Trevisan, *Comp. Meth. Appl. Mech. Eng.*, accepted.
- **Coverage in sensor networks**:
  P. Dłotko, R. Ghrist, M. Juda, MM, in preparation.

Generation of cuts

- H. J. Branin(1966), T. Weiland(1977), E. Tonti(1988) - foundations of discrete, geometric approach to Maxwell's equations as an alternative for the Galerkin method
- needed: "thick cut" (cohomology generator)
- computation of the generator based on the classical approach good only for toy examples
- real problems: more than 10^5 tetrahedra on input
- real problems solvable with coreduction algorithm

Generation of cuts



A cut for Borromean rings (254718 tetrahedra on input)

Applications

- **Lorenz equations**:
  K. Mischaikow, A. Szymczak, MM, *J. Diff. Equ.*, 2001.
- **Hénon map**:
  S. Day, R. Frongilo, R. Trevino, in preparation.
- **Kot-Schaffer map in** $L^2([-\pi, \pi])$:
  S. Day, O. Junge, K. Mischaikow, *SIAM Dyn. Syst.*, 2004.
- **Time series dynamics**:
  K. Mischaikow, J. Reiss, A. Szymczak, MM, *Phys. Rev. Lett.*, 1999.
- **Databases for multiparameter systems**:
  Z. Arai, H. Kokubu, W. Kalies, K. Mischaikow, H. Oka, P. Pilarczyk, *SIAM J. App. Dyn. Sys.*, accepted.
- **Image analysis**:
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- **Generation of cuts in electromagnetism**:
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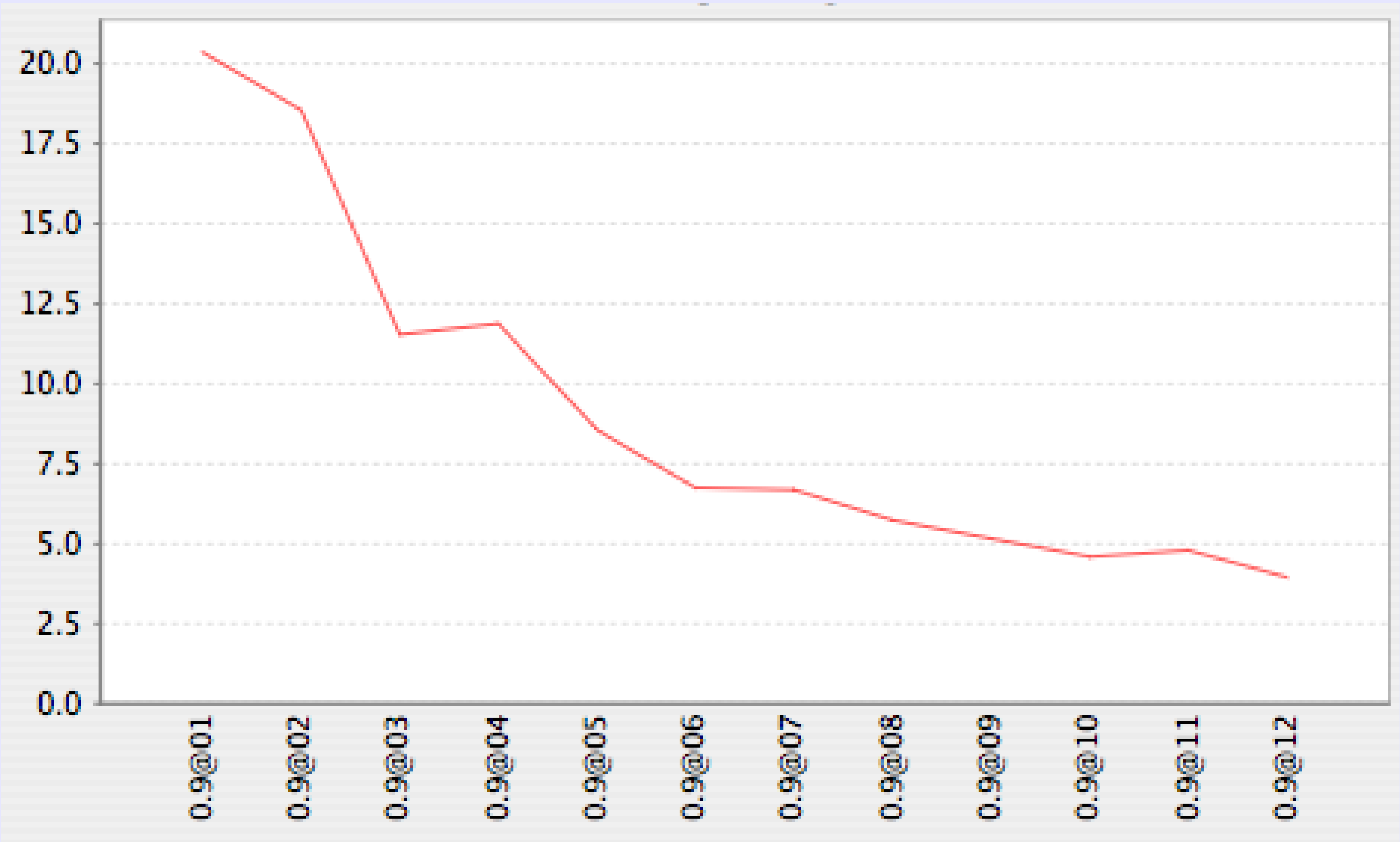
- **Coverage in sensor networks**:
  P. Dłotko, R. Ghrist, M. Juda, MM, in preparation.

Coverage in sensor networks

- V. de Silva, R. Ghrist and A. Muhammad, Blind swarms for coverage in 2-d, 2005.
- distributed computing?



Efficiency of distributed computation in a series of networks from 225 to 2700 sensors.