$\mathbb{Z}_2$-Homology of 2-manifolds may be computed in $O(n \log^*(n))$ time

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To avoid the problem with complexity, two approach can be useful:

- Reduction algorithms - after reduction the input set is small enough to use standard algorithm.
- For special input classes faster algorithms exist.

Definitions

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- S be a finite set with a gradation S_q
- $\dim s = q$ iff $s \in S_q$
- $\kappa : S \times S \to R$ s.t.

$$\kappa(s,t) \neq 0 \;\Rightarrow\; \dim s = \dim t + 1$$

Definitions

Definition

(S,κ) is an S-complex if $(R(S),\partial^\kappa)$ with $\partial^\kappa : R(S) \to R(S)$ given by

$$\partial^\kappa(s) := \sum_{t\in S} \kappa(s,t)t$$

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We assume that the coding of S carries the information about κ.

Simplicial complex

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By putting

$$\kappa(\sigma, \tau) := \begin{cases} (-1)^i & \text{if } \sigma = [A_0, A_1, \ldots A_q] \\ & \text{and } \tau = [A_0, A_1, \ldots A_{i-1}, A_{i+1}, \ldots A_q] \\ 0 & \text{otherwise.} \end{cases}$$

we obtain an S-complex whose chain complex is the classical simplicial chain complex used in simplicial homology.

Qubical complex

For every elementary cube Q and number $j \in \{1, 2, \ldots d\}$ we define the jth *nondegeneracy number* of Q by

$$\nu(Q, j) := \begin{cases} \operatorname{card}\{\, i < j \mid \operatorname{len} I_i = 1 \,\} & \text{if } \operatorname{len} I_j = 1. \\ 0 & \text{otherwise.} \end{cases}$$

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A *cubical complex* $\mathcal{C}$ in $\mathbb{R}^d$ is a finite collection of elementary cubes in $\mathbb{R}^d$ and the associated *cubical set* is the union of this collection. The cubical complex $\mathcal{C}$ has a natural gradation $(\mathcal{C}_q)_{q \in \mathbb{Z}}$, where $\mathcal{C}_q$ consists of elementary cubes of dimension q.

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$$\kappa(Q, P) := \begin{cases} (-1)^{\nu(Q,j)} & \text{if } Q = I_1 \times \cdots \times I_j \times \cdots \times I_d \\ & \text{and } P = I_1 \times \cdots \times I_j^- \times \cdots \times I_d \\ (-1)^{1+\nu(Q,j)} & \text{if } Q = I_1 \times \cdots \times I_j \times \cdots \times I_d \\ & \text{and } P = I_1 \times \cdots \times I_j^+ \times \cdots \times I_d \\ 0 & \text{otherwise.} \end{cases}$$

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An induction argument in d may be used to show that a cubical complex is an S-complex.

Subcomplexes

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\begin{aligned}
\operatorname{bd}_S A &:= \{\, t \in S \mid \kappa(s,t) \neq 0 \text{ for some } s \in A. \,\} \\
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S' \subset S & \qquad \kappa' := \kappa|_{S' \times S'}
\end{aligned}
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Theorem

If $S' \subset S$ is such that for all $t_1, t_2 \in S'$ and $s \in S$

$$s \in \operatorname{bd}_S t_1 \text{ and } t_2 \in \operatorname{bd}_S s \text{ implies } s \in S', \tag{1}$$

then (S', κ') is an S-complex.

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then (S', κ') is an S-complex.

- $S' \subset S$ satisfying (1) is *regular*
- $S' \subset S$ is *closed* in S if $\operatorname{bd}_S S' \subset S'$
- $S' \subset S$ is *open* in S if $S \setminus S'$ is closed in S

Subcomplexes

Theorem

If $S' \subset S$ is closed in S, then S' and $S \setminus S'$ are regular.

Theorem

If S' is closed in S, then

(i) $\partial^{\kappa'} = \partial^{\kappa}|_{R(S')}$

(ii) *$R(S')$ is a subcomplex of $R(S)$.*

Exact sequence

Theorem (M.Mrozek, B.Batko - 2009)

Assume $S' \subset S$ is closed in S. Let $S'' := S \setminus S'$ and $\kappa'' := \kappa|_{S'' \times S''}$. Then the inclusion and the projection

$$\begin{aligned} \iota : (R(S'), \partial^{\kappa'}) &\rightarrow (R(S), \partial^{\kappa}) \\ \pi : (R(S), \partial^{\kappa}) &\rightarrow (R(S''), \partial^{\kappa''}) \end{aligned}$$

are chain maps. Moreover, we have the following short exact sequence

$$0 \rightarrow R(S') \xrightarrow{\iota} R(S) \xrightarrow{\pi} R(S'') \rightarrow 0$$

and the following long exact sequence of homology modules

$$...H_q(R(S')) \xrightarrow{\iota_q} H_q(R(S)) \xrightarrow{\pi_q} H_q(R(S'')) \xrightarrow{\partial_k} H_{q-1}(R(S'))...$$

Relative homology

Theorem

If S' is closed in S then

$$H(R(S'')) \cong H(R(S), R(S')).$$

Reduction and coreduction pairs

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If T is a nullset of S then $H(R(S))$ and $H(R(S \setminus T))$ are isomorphic.

- (a, b) is a *reduction pair* if $\kappa(a, b)$ is invertible
- a is a *free face* of b in S if (a, b) is a reduction pair and $\operatorname{cbd}_S a = \{b\}$
- b is a *free coface* of a in S if (a, b) is a reduction pair and $\operatorname{bd}_S b = \{a\}$

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Theorem

Assume $a, b \in S$. If a is a free face of b or b is a free face of a then $\{a, b\}$ is a nullset.

Coreduction algorithm [M.Mrozek, B.Batko - 2009]

Q := empty queue;
enqueue(Q,s);
while $Q \neq \emptyset$ **do**
    s:=dequeue(Q);
    if $\text{bd}_S\, s = \{t\}$ **then**
        remove(s);
        remove(t);
        foreach $u \in \text{cbd}_S\, t$ **do**
        if $u \notin Q$ **then**
            enqueue(Q, u);
        endif;
    else if $\text{bd}_S\, s = \emptyset$ **then**
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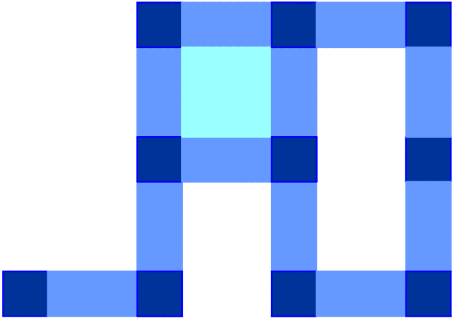
Example



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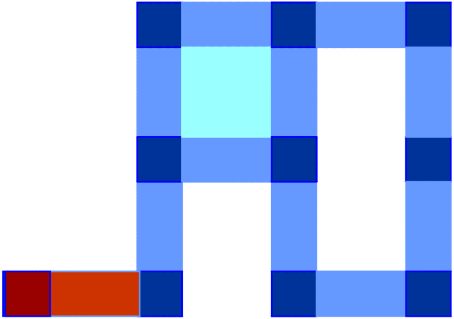

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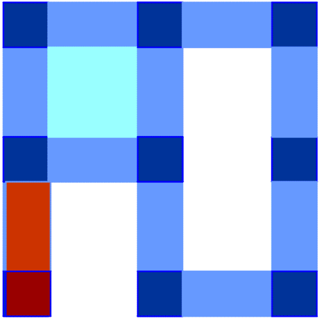

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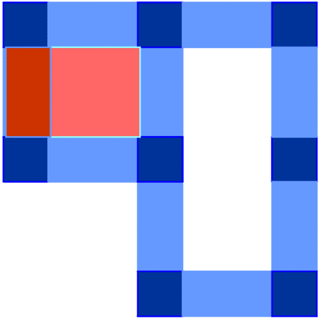

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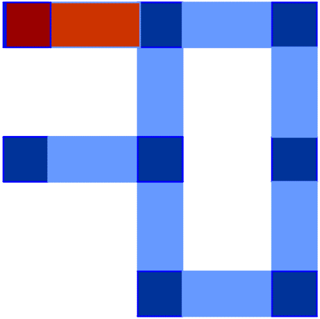

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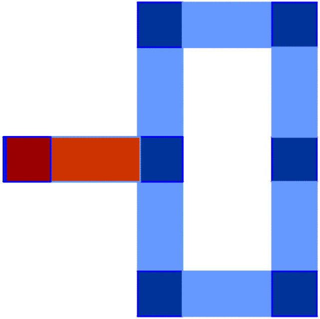

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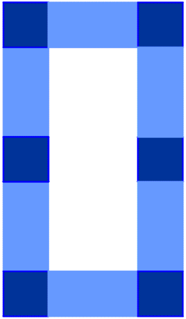

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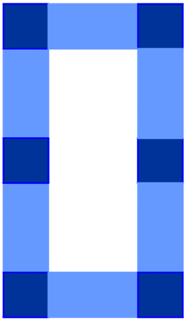

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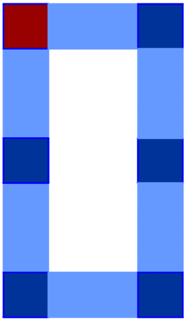

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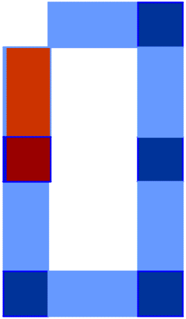

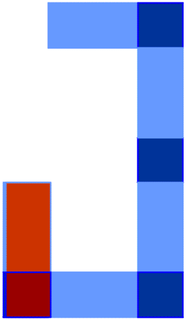

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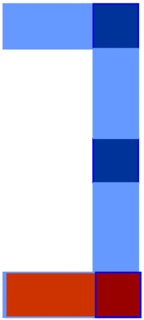

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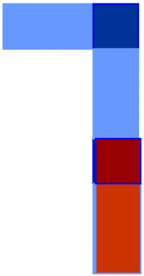

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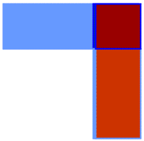

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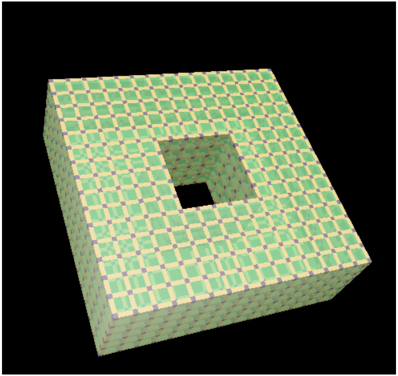

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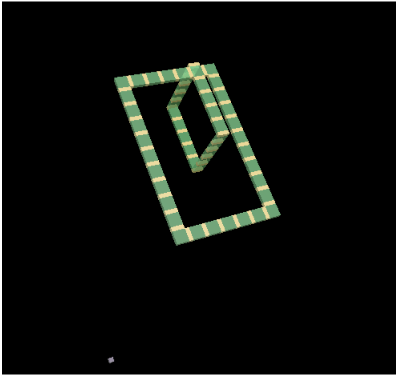

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- Experiments based on cubical sets show that coreduction homology algorithm performs much better than other algorithms.

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Definition

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Coreduction algorithm returns a vertexless S-complex.

Delfinado, Edelsbrunner algorithm

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The algorithm works as an extension for coreduction homology algorithm.

Pseudomanifold

Definition

An S-complex X is a 2-pseudomanifold *if*

- $X_q = \emptyset$ *for* $q > 2$
- *for each* $s \in X_1$ *the cardinality of* $\operatorname{cbd}_X(s)$ *is exactly two*
- *for all* $a \in X_0$ *and for all* $b_1 \neq b_2 \in X_1$ *such that* $\kappa(b_i, a) \neq 0$ *exists* $c \in X_2$ *such that* $\kappa(c, b_i) \neq 0$.

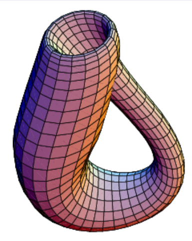
Remark

The last condition is to avoid:

2-pseudomanifold example

Example

Klein Bottle.



Example

Real projective plane.

Example

Bing's House is not a 2-pseudomanifold.

2-pseudomanifold example

This examples are not in $\mathbb{S}^3$, Delfinado-Edelsbrunner algorithm can't be used.

Connection

Definition

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Definition

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Notation

- $\mathcal{C}(X)$ the collection of connected components of X.

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- $\mathcal{C}_p(X) := \{ A \cap X_p \mid A \in \mathcal{C}(X) \}$

Standard results

Lemma

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Lemma

Let X be an S-complex with connected components

$$X^1, X^2, \ldots, X^n$$

Then

$$H(X, \mathbb{Z}_2) = \bigoplus_{i=1}^{n} H(X^i, \mathbb{Z}_2)$$

Connected 2-pseudomanifold homology

Lemma

If X is a connected 2-pseudomanifold then

$$H_2(X, \mathbb{Z}_2) = [X_2]$$

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Corollary

If X is a 2-pseudomanifold then

$$H_2(X, \mathbb{Z}_2) = \bigoplus_{A \in \mathcal{C}_2(X)} [A].$$

Main property

Theorem

Let X be a vertexless 2-pseudomanifold and let $a \in X_1$. Consider the map

$$\partial : H_2(X \setminus a, \mathbb{Z}_2) \ni [z] \to [\partial z] \to H_1(a, \mathbb{Z}_2).$$

If $\partial = 0$ then

$$H_2(X, \mathbb{Z}_2) \cong H_2(X \setminus a, \mathbb{Z}_2) \quad (2)$$

$$H_1(X, \mathbb{Z}_2) \cong \mathbb{Z}_2 \oplus H_1(X \setminus a, \mathbb{Z}_2) \quad (3)$$

If $\operatorname{im} \partial \cong \mathbb{Z}_2$ then

$$H_2(X, \mathbb{Z}_2) \cong \ker \partial \quad (4)$$

$$H_1(X, \mathbb{Z}_2) \cong H_1(X \setminus a, \mathbb{Z}_2) \quad (5)$$

Algorithm

The theorem leads to the following iterative algorithm for computing homology groups of 2-pseudomanifolds.
The Find-Union structure used in the algorithm is a standard data structure with add, find and union operations (see Cormen, Leiserson and Rivest "Introduction to algorithms").

Algorithm, [MJ, M.Mrozek - 2009]

Algorithm

```
function GetZ2Generators(S-complex X)
begin
   coreduction(X);
   S := empty Find-Union structure;
   foreach b in X_2 do S.Add(b);
   foreach a in X_1 do begin
      (b_1, b_2) := cbd(a);
      if S.Find(b_1)=S.Find(b_2) then
         S.Add(a);
      else
         S.Union(b_1, b_2);
   end;
   return S.Sets;
end;
```

Algorithm correctness

Theorem (MJ, M.Mrozek - 2009)

The algorithm called with a 2-pseudomanifold X returns a collection of sets $\mathcal{S}$ such that

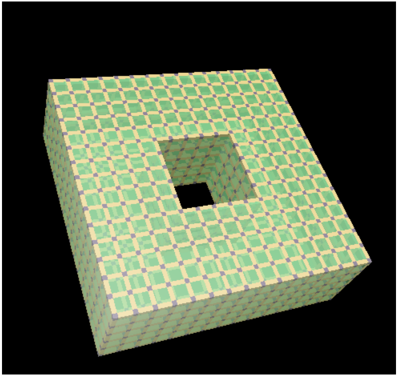
$$H(X, \mathbb{Z}_2) = \bigoplus_{S \in \mathcal{S}} \sum_{s \in S} [s].$$

Algorithm complexity

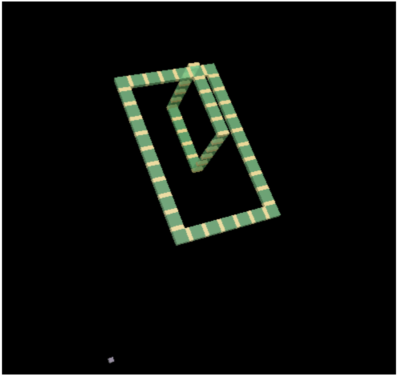
Theorem (MJ, M.Mrozek - 2009)

The algorithm runs in $O(n \log^*(n))$ time, where n denotes the cardinality of the S-complex on input.

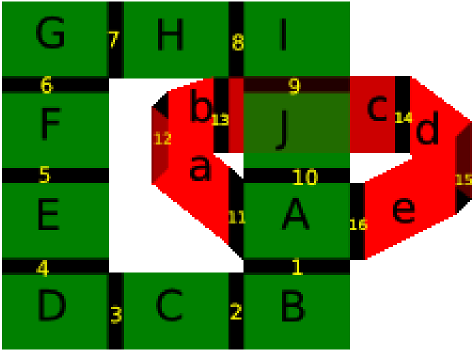
Torus S-complex example



Torus S-complex example

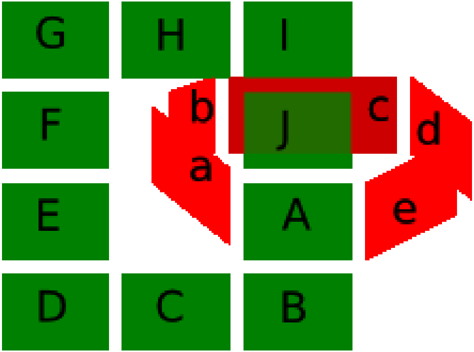
Vertexless reduced torus: 

Torus S-complex example



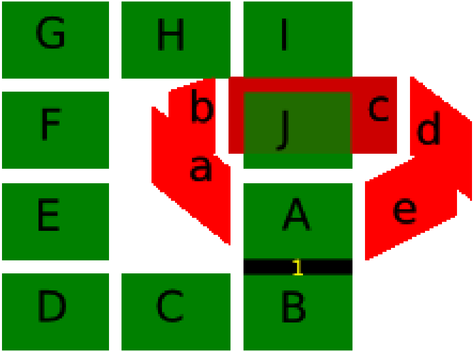
$$
\begin{aligned}
X_0 &= \emptyset \\
X_1 &= \{1, 2, \ldots, 16\} \\
X_2 &= \{A, B, \ldots, J, \\
&\quad a, b, \ldots, e\}
\end{aligned}
$$

Initial step



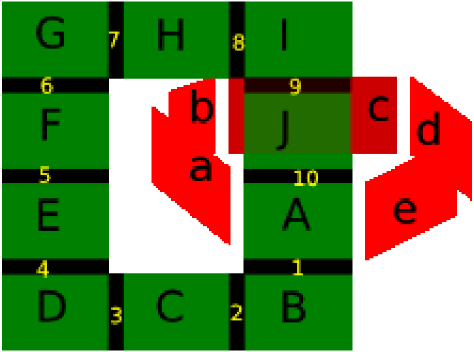
$$S_1 = \emptyset$$
$$S_2 = \{\{A\}, \{B\}, \ldots, \{J$$
$$\{a\}, \{b\}, \ldots, \{e\}\}$$

Considered element 1



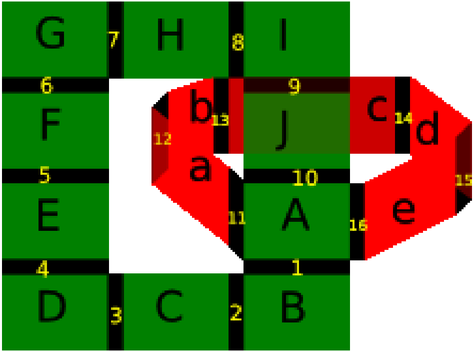
$$
\begin{aligned}
S_1 &= \emptyset \\
S_2 &= \{\{A, B\}, \ldots, \{J\} \\
&\quad \{a\}, \{b\}, \ldots, \{e\}\}
\end{aligned}
$$

Considered element 10



$$
\begin{aligned}
S_1 &= \{\{10\}\} \\
S_2 &= \{\{A, B, \ldots, J\} \\
&\quad \{a\}, \{b\}, \ldots, \{e\}\}
\end{aligned}
$$

Considered element 16



$$
\begin{aligned}
S_1 &= \{\{10\},\{16\}\} \\
S_2 &= \{\{A, B, \ldots, J, \\
&\quad a, b, \ldots, e\}\}
\end{aligned}
$$

Pseudomanifold limitation

It is obvious from coreduction algorithm we cannot get pseudomanifold for each input data:

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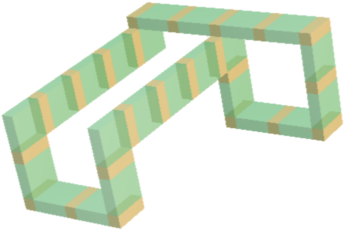
Reduced Bing's house:



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In coreduction algorithm we use empty set as a boundary for vertices. It gives a possibility to start coreductions, because we could have first pair for coreduction. When coreduction algorithm ends, the input S-complex doesn't have any vertices. We could remove an edge and start the algorithm again. Work on this is in progress. The main problem is in time complexity for getting generators.

References

- one space homology theory with compact supports for locally compact sets (Steenrod 1940, Massey 1978)
- Cecil Jose A. Delfinado, Herbert Edelsbrunner "An incremental algorithm for Betti numbers of simplicial complexes" (Computer Aided Geometric Design, 1995)
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- M. Mrozek, B. Batko "Coreduction Homology Algorithm" (Discrete and Computational Geometry, 41(2009), 96-118.)
- MJ, Mrozek, "$\mathbb{Z}_2$-Homology of 2-manifolds may be computed in $O(n \log^*(n))$ time", in preparation.